# Summation Calculation of Delayed Neutron Emission and its Application to Reactor Kinetics

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## ABSTRACT

The kinetics of a point reactor is solved directly form buildup and decay of fission-product (FP) nuclei for the first time. The purpose of this paper is to identify possible sources of the peculiar behavior of inhour equations calculated from delayed neutron temporal data in ENDF/B-VI, that were obtained from FP fission yields decay data. Specifically, the inhour equation is calculated in the summation method directly from FP data in ENDF/B-VI. For $^{235}U$, $^{238}U$ and $^{239}Pu$, the present calculation of the inhour equation show similar behavior to those obtained from the temporal data. To identify FP data responsible for this peculiarity, the asymptotic form of the inhour equation at infinitely long periods is examined. It is found that the most important precursors for long reactor periods are found $^{137}I$, $^{88}Br$ and $^{87}Br$. They cover more than 60 % of the reactivity. Among them, $^{137}I$ alone covers 30-50 %. Moreover, $^{136}Te$ is found a possible candidate for the peculiarity from the time dependence of the delayed neutron emission. It is recommended that the precision of their Pn values should be improved experimentally. For $^{137}I$, $^{88}Br$, and $^{87}Br$, the precision, dPn/Pn, should be decreased down to 2 % and for $^{136}Te$ to 5%. Except for the Pn and fission yield data of these precursors, the presently available FP data seem to provide reasonable time dependence of DN emission having almost comparable precision ($\leq$ 10%) with empirical evaluations.

## 1. INTRODUCTION

The time dependence of the delayed neutron (DN) emission is a crucial parameter to determine the reactor kinetics. The delayed neutrons are emitted from various fission-product (FP) nuclei after β decays of their parent nuclei. In principle, the time dependence of the DN emission can be calculated in the summation method using FP fission yields and decay data. However, this line of study was never taken seriously because the availability and precision of the FP data was far from being satisfactory.

The DN emission property that determines kinetics of a point reactor is the time dependence of delayed neutrons emitted per unit time after a pulse fission, $n_d(t)$. This quantity is related to the total number of delayed neutrons emitted after a pulse fission, $\nu_d$, given by

$$\nu_d = \int_0^\infty dt\, n_d(t) \quad (1)$$

Hereafter, $n_d(t)$ is referred to as the DN activity.

Delayed neutron six-group temporal parameter sets in the ENDF/B-VI are the first and only ones that are obtained from the summation calculations. The DN activity is approximated with a six-group temporal set as

$$n_d(t) \approx \nu_d \sum_{i=j}^{6} \alpha_j \lambda_j \exp\left(-\lambda_j t\right) , \tag{2}$$

$$\sum_{j=1}^{6} \alpha_j = 1 . \tag{3}$$

In ENDF/B-VI, the decay constant, $\lambda_i$, and fractional yield, $\alpha_i$, are determined to fit a summation calculation of $n_d(t)$ for the fast fission using the FP data in a preliminary version of ENDF/B-VI. As for the average number of delayed neutrons per fission $\nu_d$, END/F-VI adopted conventional values that were not obtained from the summation calculation. It is noted that the temporal parameter set ($\lambda_i$, $\alpha_i$) is assumed energy independent while the $\nu_d$ value is energy dependent.

Unfortunately, the inhour equation obtained from the ENDF/B-VI temporal parameter set shows a peculiar behavior [1] compared with the temporal set of contemporary use by Tuttle[2]. **Figure 1** compares the inhour equations obtained from the two temporal sets, and shows striking difference in the reactivity for long reactor periods. In this figure, there is no neutron energy dependence because the reactivity in dollar (ρ$) is written only with the temporal parameters, $\lambda_i$'s and $\alpha_I$'s, and independent of $\nu_d$.

In the following, we attempt to identify sources of the difference at long period, focusing on the asymptotic behavior of the kinetics of a point reactor. Specifically, the inhour equation is calculated and analyzed directly from the FP yields and decay data in ENDF/B-VI. Similarly to the FP decay heat and DN activity calculations, this calculation is referred to as the summation calculation of the reactor kinetics.

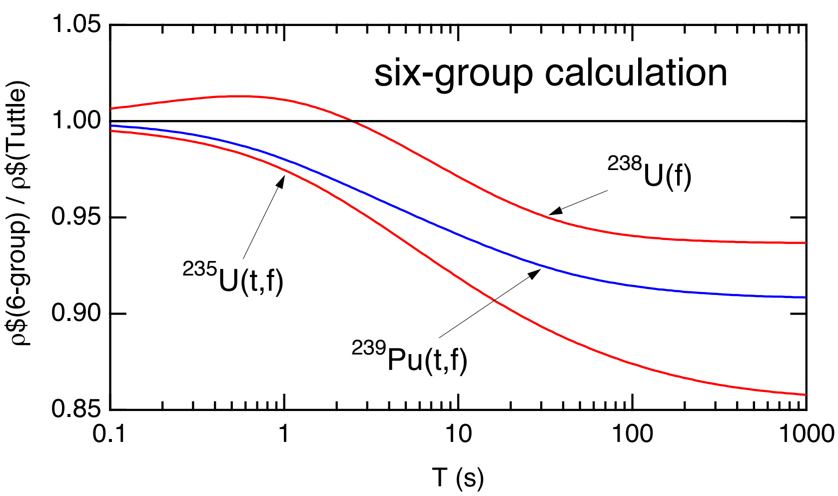


**Figure 1.** The inhour equation ρ$($T$) calculated with the temporal parameter sets in ENDF/B-VI and Tuttle's recommendation. The symbols t and f in the parentheses denote the thermal and fast neutrons, respectively.

## 2. Kinetics of a point reactor

We consider a point reactor that was critical before $t < 0$. The reactor power $p(t)$ after a step insertion at $t$=0 is obtained from the following integro-differential equation.

$$\frac{dp}{dt} = \frac{\rho(t) - \beta}{\Lambda} p(t) + \int_{-\infty}^{t} du \frac{p(u)}{\Lambda} \frac{n_d(t-u)}{\nu} \tag{4}$$

$\nu$ : total number of neutron emitted after a pulse fission

$\beta = \nu_d/\nu$ : DN fraction

$\Lambda$ : mean neutron generation time

$\rho$ : reactivity

Here, the $\nu$ value is known precisely enough (almost independently of uncertainty in the $\nu_d$ value because $\nu_d/\nu(=\beta)<<1$). The DN activity, $n_d(t)$, is calculated from FP fission yields and decay data in the summation method. Formally Eq. (4) can be solved using Laplace transform.
The inhour equation obtained from from Eq. (4) gives a relation between the reactivity in dollar ($\rho\$$) and the asymptotic reactor period ($T$).

$$\rho\$ = \frac{\int_0^{\infty} dt \exp\left(-t/T\right) m_d(t)/\nu_d}{\nu T} \tag{5}$$

$$m_d(t) = \int_{-\infty}^{0} du\, n_d(t-u) \tag{6}$$

$$\nu_d = m_d(0) = \int_0^{\infty} dt\, n_d(t) \tag{7}$$

In the following, the inhour equation obtained from a summation calculation of $n_d(t)$ is referred to as the inhour equation in the summation method.
It is useful to consider an asymptotic formula of Eq. (5) for a long $T$ ($T\rightarrow\infty$). It is given by

$$\rho\$ \rightarrow \frac{N_d}{\nu_d} \frac{1}{\nu T} \tag{8}$$

$$N_d = \int_0^{\infty} dt\, m_d(t) \tag{9}$$

The asymptotic behavior is determined by $N_d/\nu_d$. Both $N_d$ and $\nu_d$ are functionals of $n_d(t)$, and can be calculated readily in the summation method with help of Eqs. (6), (7) and (9). It is also noted that $\nu_d$ is explicitly contained in $\rho\$$ (Eqs. (5) and (8)) although it is canceled out in the temporal parameter approximation (2).

## 3. Inhour equation in the summation method

The inhour equation is calculated in the summation method with the FP data in ENDF/B-VI. The numerical calculation of $n_d(t)$ and $m_d(t)$ are performed with a small computer program developed by the present author [3]. In the present study, the original value of the independent fission yield of $^{86}$Ge is divided by 100 for the thermal fission of $^{235}$U because it is unrealistically large due to a typographical error [4].

**Figure 2** shows the inhour equation in the summation method. The neutron energy dependence in the figure arises from the fission yields. It has the same feature as the one in **Fig. 1**; the asymptotic ρ\$($T$) behavior for a large $T$ is significantly different from Tuttle's except for $^{239}$Pu.

To examine the asymptotic behavior in detail, **Fig. 3** shows values of the coefficient $N_d/\nu_d$ in Eq. (8), together with $\nu_d$ and $N_d$ values. These values are calculated in the summation method. For $^{235}$U(t,f), $^{238}$U(f) and $^{239}$Pu(t), the $N_d/\nu_d$ and $N_d$ values are significantly different from Tuttle's. These differences should be compared with the asymptotic behavior in **Fig. 1**. It is reasonable to assume that major sources of the peculiar ρ\$($T$) behavior are common in **Figs. 1** and **2** at least for $^{235}$U(t,f), $^{238}$U(f), and $^{239}$Pu(t).

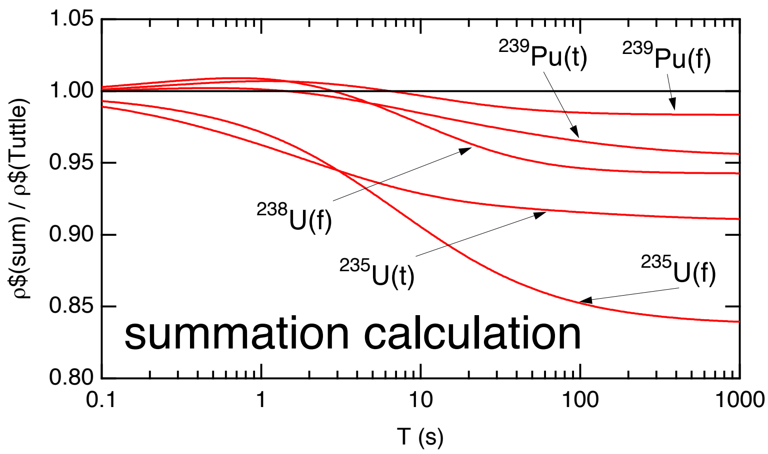


**Figure 2.** The inhour equation calculated in the summation method. The symbols in the Figure are the same as those in Fig. 1.

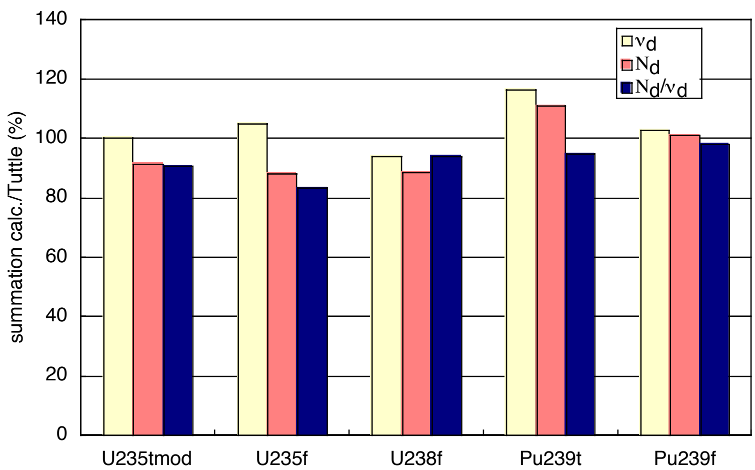


**Figure 3.** Comparision of the $N_d/\nu_d$, $N_d$, and $\nu_d$ value in the asymptotic formula (Eq. (8)) between the summation calculation and Tuttle's six-group approximation.

## 4. Key precursors for kinetics of a point reactor

From Eqs. (5) and (8), it is reasonable to assume that key precursors to the asymptotic ρ$ behavior give large contributions to $N_d$. **Figure 4** clearly shows that $^{137}$I, $^{88}$Br and $^{87}$Br are dominantly important covering more than 60 % of the $N_d$ value, although their contributions to $\nu_d$ are relatively small. It is also noteworthy that the most important precursor is $^{137}$I although it has been believed that the longest-lived $^{87}$Br is the most important at a long period.

One might further identify a DN precursor relevant to the ρ$ behavior from the time dependence of $m_d(t)$ in Eq. (5). A simple and intuitive way is to assume that the difference between the summation calculation and Tuttle's,

$$\Delta m_d(t) = m_d^{\mathrm{Tuttle}}(t) - m_d^{\mathrm{SUM}}(t), \tag{10}$$

is caused mainly by an erroneous FP data of a single DN precursor with half life λ. Then, $\Delta m_d(t)$ is proportional to $\exp(-\lambda t)$. If we notice that the function $t\times\exp(-\lambda t)$ has a peak at $t=1/\lambda$, the peak time of $t\times\Delta m_d(t)$ correspond to the lifetime of the relevant precursor, whose half life is $(\ln 2)/\lambda$.

For $^{235}$U(f) in **Fig. 5**, $t\times\Delta m_d(t)$ has a peak at 36.7 s (its corresponding half life 25.4 s). It is also seen clearly that this difference is fitted well by $^{137}$I (half life 24.5 s). For $^{238}$U(f), the obtained half life is 20.5 s. In this case, $^{88}$Br (16.5 s) and/or $^{136}$Te (17.5 s) as well as $^{137}$I can account for the difference. Here, $^{136}$Te is taken into account because it also gives an appreciable contribution to $N_d$. Similarly, it is found that, for $^{235}$U(t) and $^{239}$Pu(t), $\Delta m_d(t)$ can be fitted by $^{88}$Br and/or $^{136}$Te.

The above argument seems reasonable because the adjustment of $m_d(t)$ by the residual term obtained from the fitting significantly improves the inhour equation as shown in **Fig. 6**. If we notice that Tuttle's evaluation may include a margin of about 5%, it is not too optimistic to conjecture that the summation calculation is now close to being practical.

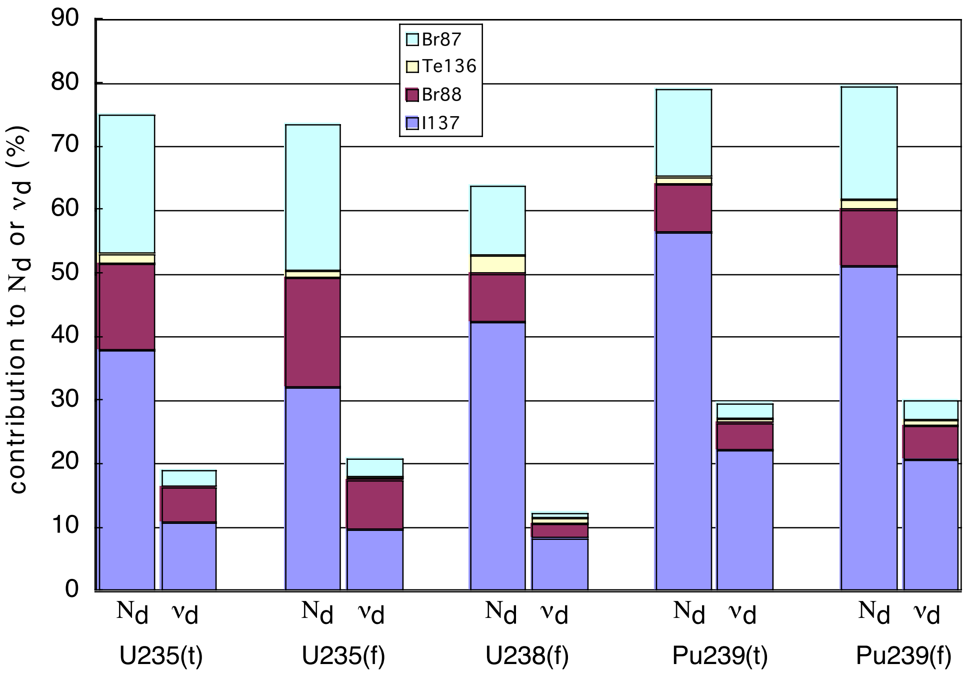


**Figure 4.** Contributions to $N_d$ and $\nu_d$ values from individual precursors after a pulse fission. The corresponding precursors in a bar are $^{137}$I, $^{88}$Br, $^{136}$Te and $^{87}$Br from the bottom.

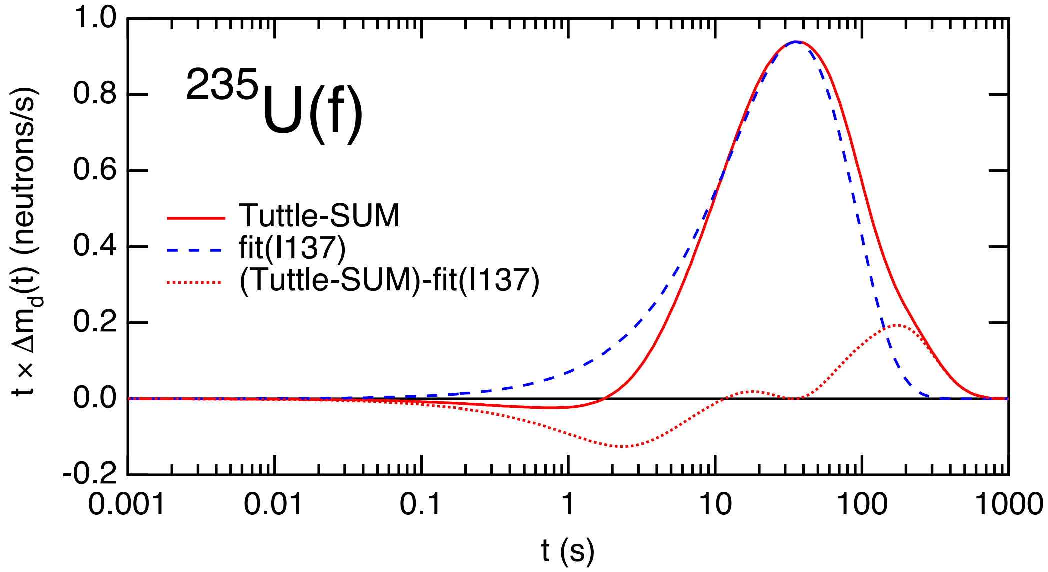


**Figure 5.** The difference of $m_d(t)$ between the summation calculation and Tuttle's approximation. Shown are $t \times \Delta m_d(t)$ and its fitting ((fitting constant)$\times t \times \exp(-\lambda t)$) with $^{137}$I.

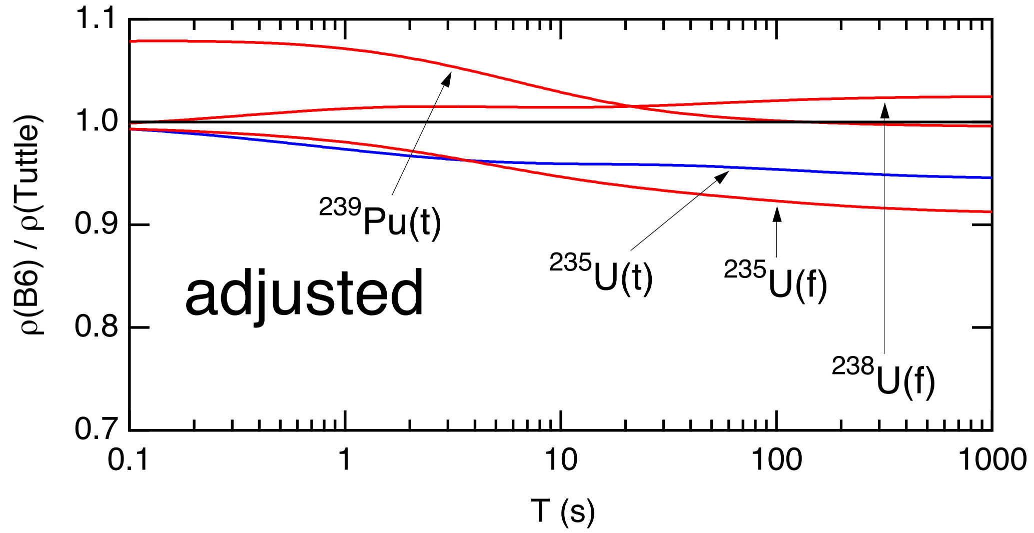


**Figure 6**. The adjusted inhour equation. In this adjustment, the original $m_d(t)$ is supplemented by the residual contribution obtained from fitting $\Delta m_d(t)$.

## 5. Precision of fission yields and decay data

The contribution to $m_d(t)$ from each of the above precursors is approximately given by Pn$\times Y \times \exp(-\lambda t)$ where $Y$ and Pn denote the cumulative fission yield and the DN emission probability of the precursor, respectively. Further consideration is needed to identify which precursor data is relevant to the

peculiar $m_d(t)$ (and equivalently ρ$) behavior.
Fortunately, the half lives of the above precursors are sufficiently precise (about 1% or better) so that the argument in the previous chapter works well.
Let us consider a measure of the required precision of the Pn and Y values. Required precision of $n_d(t)$, and therefore $m_d(t)$, is evaluated to be 5 % [4]. If we assume that the error is due to a single precursor,

$$(\mathrm{dPn/Pn})^2 + (\mathrm{d}Y/Y)^2 = (5\%)^2. \tag{11}$$

Since the fission yields are more difficult to evaluate precisely, it is reasonable to require that

$$\mathrm{dPn/Pn} = 3\%, \qquad \mathrm{d}Y/Y = 4\%. \tag{12}$$

As for Pn values in ENDF/B-VI, unfortunately, the relative precision, dPn/Pn, is still large. It is 6 % even for the most important $^{137}$I. Uncertainties of the fission yields are much larger in ENDF/B-VI.
It is recommended to improve the precision of Pn values of $^{137}$I, $^{88}$Br, $^{87}$Br, and $^{136}$Te because precise Pn measurements can now be performed at some laboratories. For the most important precursors $^{137}$I, $^{88}$Br and $^{87}$Br, the precision should be better than 3%. They should be at least as good as 2 % to eliminate uncertainties from the Pn values. For $^{136}$Te, the precision of about 5% can be sufficient because this precursor has a relatively smaller contribution to $N_d$ than the others.
Once we obtain the precise Pn values, it is much easier to improve the precision of fission yield values because Pn values are branching ratios of decay chains. Hence, the precise Pn measurement is the first step to make the summation calculation practical for the reactor kinetics calculation.

## CONCLUSIONS

In this paper, the kinetics of a point reactor is examined directly from FP fission yields and decay data in ENDF/B-VI for the first time. The inhour equation shows peculiar underestimate of reactivity at long periods. It is quite similar to the one obtained from the DN temporal data in ENDF/B-VI. $^{137}$I, $^{88}$Br and $^{87}$Br are found to cover 60-80% of the reactivity at the infinitely long period. Among them, $^{137}$I is the most important. From an additional analysis on the time dependence of the DN activity, we conclude that $^{137}$I, $^{88}$Br and $^{136}$Te could cause the peculiarity of the inhour equation. In order to improve the summation calculation of the reactor kinetics, it is recommended that the precision of the Pn value should be improved to 2 % for the most important $^{137}$I, $^{88}$Br and $^{87}$Br, and to 5 % for $^{136}$Te. Except for the Pn and/or fission yield data of these precursors, the presently available FP data seem to provide reasonable time dependence of DN emission having almost comparable precision ($\leq$ 10%, see Fig. 6) with empirical evaluations. Hence, we believe that it will not take long time to realize the summation technique as a practical way to evaluate DN data.

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