APPLICATION OF VARIANCE-TO-MEAN METHOD TO ACCELERATOR-DRIVEN SUBCRITICAL SYSTEM

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ABSTRACT

To apply the variance-to-mean method to the pulse-mode operation of the accelerator-driven subcritical reactors, a new formula for the repetition injection of pulsed neutrons has been derived based on the Master equation. Its validity has been confirmed by the numerical simulation and the experiment performed at the Kyoto University Critical Assembly. As a result, in spite of the ADS operation modes, namely the steady state or the pulse injection, the variance-to-mean method is applicable to evaluate the prompt neutron decay constant as a measure of subcriticality.

1. INTRODUCTION

Recently many research activities are concentrated on the Accelerator-Driven Subcritical System (ADS) as a candidate of an innovative nuclear reactor in the next generation. This system consists of a high current proton accelerator and a subcritical nuclear reactor. A large number of neutrons generated by spallation reaction are supplied into the subcritical reactor. Even though the ADS reactor is operated at subcritical state, we need to monitor subcriticality continuously, in order to operate it safely without depending on multiple reactivity control devices. Then development of the subcriticality monitor is an important issue.

The variance-to-mean method (V-to-M) is the conventional one for estimating nuclear reactor kinetic parameters. The V-to-M ratio of neutron counts detected during a certain gate width contains the prompt neutron decay constant as an index of subcriticality. To apply the V-to-M method to ADS reactors as the subcriticality monitor, one of the authors have already derived the V-to-M formula with the spallation neutron source[1,2].

In this paper we concentrate on another problem on the application of V-to-M method to ADS. The previous formula was derived for a steady neutron source. Then the spallation neutron source for the ADS may be a pulsed-neutron source, because the pulse mode operation of proton accelerator is planned. Hence it is worth while investigating whether or not the V-to-M method can be extended for the pulsed neutron source. Moreover the extension of V-to-M method to a quasi-steady state, namely the repetition of the same temporal behavior, is interesting from the theoretical viewpoint. Therefore we tried to derive a new V-to-M formula for the pulsed-neutron source, and investigated its validity through the numerical simulation and the experiment performed at the Kyoto University Critical Assembly (KUCA).

2. DERIVATION

We assumed for simplicity both the one-point reactor model and the neutron diffusion model without delayed neutrons. The outline of the derivation is as follows; (1) the Master equation taking into account of the pulsed spallation neutron source is derived, and then (2) it is solved by the generating function technique.

To obtain the V-to-M formula for a quasi-steady state realized by the repetition injection of pulsed neutrons, we use the following general expression as the pulsed neutron source.

$$S(t) = \sum_{L=0}^{J(t)-1} \delta(t - LT) \tag{1}$$

where T denotes a pulse period, and J(t) denotes the number of pulsed neutron bursts injected into the system investigated up to the time t. Here we assume that an injection time of a pulsed neutron, after which the system keeps the quasi-steady state, is chosen to be the time origin. This pulsed neutron burst is generated by the spallation reaction, and one spallation reaction generates n-neutrons with the probability q(n) at a time.

The following Master equation for the repetition injection of pulsed neutrons can be described. This equation is the same as one for the previous one derived by I. Pazsit and Y. Yamane [1,2], except for the description of neutron source.

$$\begin{aligned}\frac{d}{dt}P(N,Z,t) = &\sum_n P(N-n,Z,t)q(n)S(t) + \lambda_c P(N+1,Z,t)(N+1) \\ &+ \lambda_d P(N+1,Z-1,t)(N+1) + \sum_n \lambda_f P(N+1-n,Z,t)(N+1-n)p(n) \\ &- P(N,Z,t)\left[S(t) + N(\lambda_c + \lambda_d + \lambda_f)\right]\end{aligned} \tag{2}$$

where P(N,Z,t) represents the probability that at time t there are N(t) neutrons and Z(t) counts, which is the cumulative neutron counts from the time origin up to t. The probability p(n) means that n neutrons are generated by one fission reaction. The notations of λ_c, λ_d, and λ_f are the transition probabilities for neutron capture, neutron detection, and fission process, respectively. By introducing the following probability generating functions,

$$F(x,z,t) = \sum_N \sum_Z x^N z^Z P(N,Z,t) \tag{3}$$

$$f(x) = \sum_n x^n \, p(n) \tag{4}$$

$$g(x) = \sum_n x^n \, q(n) \tag{5}$$

The following equation is readily obtained.

$$\frac{\partial F(x,z,t)}{\partial t} = F(x,z,t)S(t)(g(x)-1) + \frac{\partial F(x,z,t)}{\partial x}\left\{\lambda_c(1-x) + \lambda_d(z-x) + \lambda_f(f(x)-x)\right\} \tag{6}$$

From the above equation, two coupled-equations concerning two first moments N(t) and Z(t) can be derived by means of the conventional treatment for the probability generating function,

$$\frac{dN(t)}{dt} = -\alpha N(t) + \bar{q}\, S(t) \tag{7}$$

$$\frac{dZ(t)}{dt} = \lambda_d \, N(t) \tag{8}$$

where the reactivity, ρ, the neutron generation time, Λ, and the prompt neutron decay constant, α, are defined by

$$\rho = \frac{\bar{\nu}\lambda_f - (\lambda_c + \lambda_f + \lambda_d)}{\bar{\nu}\lambda_f} \tag{9}$$

$$\Lambda = \frac{1}{\bar{\nu}\lambda_f} \tag{10}$$

$$\alpha = -\frac{\rho}{\Lambda} \tag{11}$$

In these definitions, $\bar{\nu}$ and $\bar{q}$ represent the number of mean neutrons generated by a fission and a spallation reaction, respectively. The neutron density N(t) can be solved by the Laplace transformation of Eq.(7) as follows:

$$N(t) = \bar{q} \sum_{L=0}^{J(t)-1} e^{-\alpha(t-LT)} \tag{12}$$

We note that the magnitude of neutron density is proportional to the number of mean neutrons, $\bar{q}$, generated by a spallation reaction. Hence an intense spallation neutron source is needed to realize a high flux reactor. Substituting Eq.(12) into Eq.(8) and solving it by the Laplace transformation, we can get the following solution of the cumulative neutron counts Z(t).

$$Z(t) = \frac{\lambda_d \bar{q}}{\alpha} \sum_{L=0}^{J(t)-1} \left\{1 - e^{-\alpha(t-LT)}\right\} \tag{13}$$

where we use the initial condition that there are no neutron counts at the time origin. As might be expected, the cumulative neutron counts saturate when the time elapsed enough after the first neutron injection corresponding to the time origin.

To obtain the second moments by the conventional treatment, the following modified second moments are introduced.

$$\mu_{NN}(t) = \langle N^2(t)\rangle - \langle N(t)\rangle^2 - \langle N(t)\rangle \tag{14}$$

$$\mu_{ZZ}(t) = \langle Z^2(t)\rangle - \langle Z(t)\rangle^2 - \langle Z(t)\rangle \tag{15}$$

$$\mu_{NZ}(t) = \langle N(t)Z(t)\rangle - \langle N(t)\rangle\langle Z(t)\rangle \tag{16}$$

Then three coupled-equations including the above modified second moments are obtained from Eq.(6),

$$\frac{d\mu_{NN}(t)}{dt} = -2\alpha\mu_{NN}(t) + \lambda_f \overline{\nu(\nu-1)}N(t) + \overline{q(q-1)}S(t) \tag{17}$$

$$\frac{d\mu_{NZ}(t)}{dt} = -\alpha\mu_{NZ}(t) + \lambda_d \mu_{NN}(t) \tag{18}$$

$$\frac{d\mu_{ZZ}(t)}{dt} = 2\lambda_d \mu_{NZ}(t) \tag{19}$$

where the second moments of the fission and the spallation reactions are used.

$$\overline{\nu(\nu-1)} = \left.\frac{\partial^2 f(x)}{\partial x^2}\right|_{x=1} = \sum_n n(n-1)p(n) \tag{20}$$

$$\overline{q(q-1)} = \left.\frac{\partial^2 g(x)}{\partial x^2}\right|_{x=1} = \sum_n n(n-1)q(n) \tag{21}$$

Solving the coupled-equation, Eqs.(17)(18) and (19), we can get $\mu_{zz}(t)$.

$$\mu_{ZZ}(t) = 2\lambda_d^{\ 2} \sum_{L=0}^{J(t)-1} \left\{ a + b\,e^{-2\alpha(t-LT)} + c\,e^{-\alpha(t-LT)} + d\,(t-LT)e^{-\alpha(t-LT)} \right\} \tag{22}$$

where four new notations are defined as follows:

$$a = \frac{\lambda_f \bar{q}\,\overline{\nu(\nu-1)} + \alpha\,\overline{q(q-1)}}{2\alpha^3} \tag{23}$$

$$b = \frac{-\lambda_f \bar{q}\,\overline{\nu(\nu-1)} + \alpha\,\overline{q(q-1)}}{2\alpha^3} \tag{24}$$

$$c = -\frac{\overline{q(q-1)}}{\alpha^2} \tag{25}$$

$$d = -\frac{\lambda_f \bar{q}\,\overline{\nu(\nu-1)}}{\alpha^2} \tag{26}$$

By using the solutions, Eqs.(13) and (22), we can obtain the V-to-M formula for the repetition injection of pulsed neutrons as follows [3],

$$Y(t) = \frac{2\lambda_d \alpha}{\bar{q}} \left\{ \frac{\sum_{L=0}^{J(t)-1} \left[a + b\,e^{-2\alpha(t-LT)} + c\,e^{-\alpha(t-LT)} + d\,(t-LT)e^{-\alpha(t-LT)} \right]}{\sum_{L=0}^{J(t)-1} \left[1 - e^{-\alpha(t-LT)} \right]} \right\} \tag{27}$$

The comparison of the Y-value between the pulsed source with the pulse period of 10 ms and the steady source is illustrated in Figure 1. These examples were calculated by using the parameters tabulated in Table I. We can see from Figure 1 that the Y-curve fluctuates corresponding to the pulse period. This is the significant characteristic of the V-to-M for the repetition injection of pulsed neutrons.

Table I Parameters used for numerical examples

$\bar{q}$	40	$\bar{\nu}$	2.416
$D_q = \overline{q(q-1)} / (\bar{q})^2$	3	$D_\nu = \overline{\nu(\nu-1)} / (\bar{\nu})^2$	0.7978
λ_f [s^{-1}]	1.524E4	λ_d [s^{-1}]	1.522
$\varepsilon = \lambda_d / \lambda_f$	1.0E-4	α [s^{-1}]	390

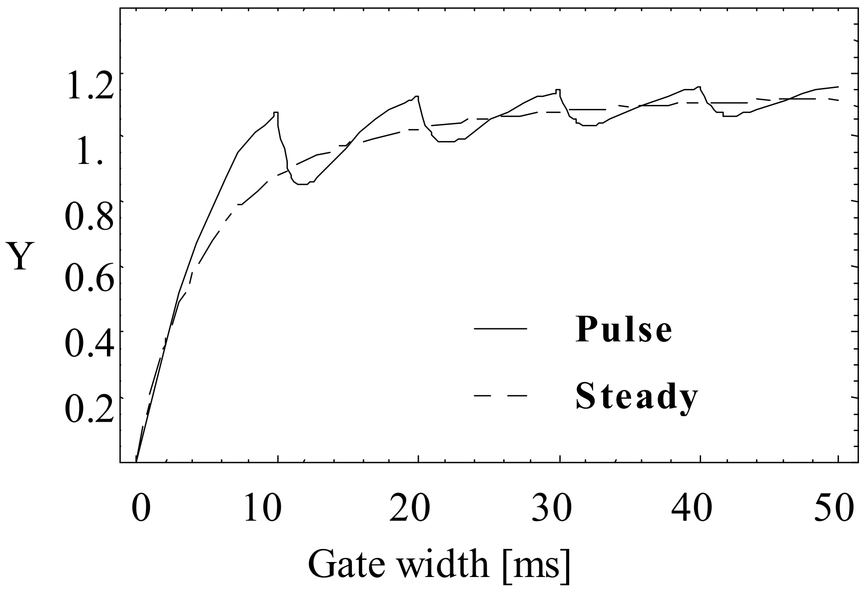

Figure 1. Comparison of Y-value between pulsed and steady source.

3. SIMULATION AND EXPERIMENT

3.1 SIMULATION

The Monte-Carlo simulation was performed in a simplified cylindrical system having the radius of 18 cm and the height of 40 cm. The pulsed neutron source is located at the center of the system. The whole side of the system was assumed to be the neutron detector, namely all neutrons out-going through the side were registered as neutron counts. Neutrons injected into the system with a period T were traced based on the neutron diffusion model with four energy-groups until they disappeared by absorption or leakage. Then a time series data of neutron counts was accumulated, and the Y-value was calculated.

The system parameters except for the prompt neutron decay constant, α, are same as Table I. In this simulation, the prompt neutron decay constants were chosen as 2400, 4100 and 7600 s^{-1}. In the high subcriticality system with a large α value, good statistics of Monte-Carlo calculation is attained in a relatively short calculation time due to a short length of fission chain. The result corresponding to the pulse period of 2 ms is illustrated in Figure 2. The notations, Low, Medium and High, mean the degree of subcriticality. Even though the pulse period and the prompt neutron decay constants are not identical to ones in Figure 1, these results of simulation reproduce well the characteristic of the theoretical prediction, namely the Y-value fluctuation synchronizing with the pulse period.

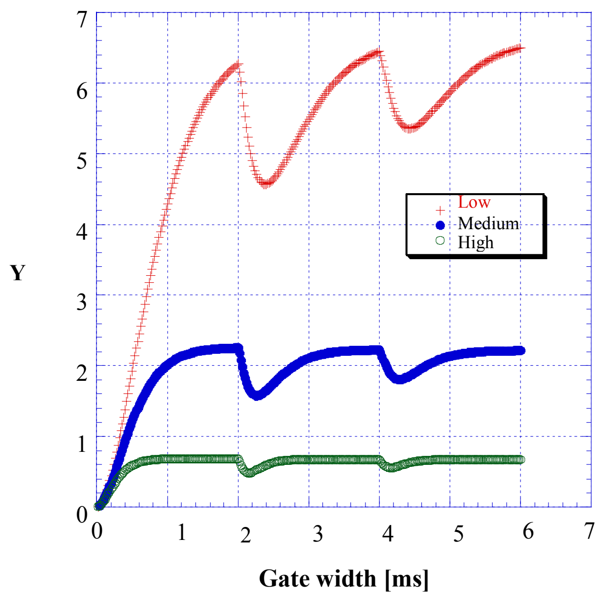

Figure 2. Y-values calculated by Monte-Carlo method.

3.2 EXPERIMENT

To confirm theoretical and numerical studies, moreover, we performed experiments at the polyethylene-moderated enriched-uranium core of KUCA. The schematic plan view of the experimental system is depicted in Figure 3. The core consists of two regions, namely fuel region and reflector region. Control and safety rod channels are positioned at boundary of the two regions. The fuel region is composed of fuel element, which contains about 40 cm long fuel part and about 50 cm long polyethylene reflectors at both ends. The fuel part is composed of 36 cells. A unit cell is made of a 93.5% enriched-uranium aluminum alloy plate of 1/16” thickness and a polyethylene moderator plate of 3/8” thickness. These materials are packed in a 5.43 cm square and 152.4 cm long aluminum tube. The reflector element contains only polyethylene blocks in the same aluminum tube. The subcriticalities of the system were chosen as 1.7$ and 3.6$ by controlling the elevation of control and safety rods.

The tritium-target of Cokcroft & Walton accelerator to generate pulsed neutrons by D-T reaction is positioned outside the reflector region. Three BF3 detectors are located at the center of fuel region, D2, and the boundaries of fuel and reflector regions, D1 and D3, as shown in Figure 3. The signals from each detector are fed into a multi-channel scaler operated with 250 micro second channel-width. The data accumulation of multi-channel scaler starts at the time synchronized with the neutron pulse injection. A time series data of neutron counts was accumulated, and Y-value was calculated.

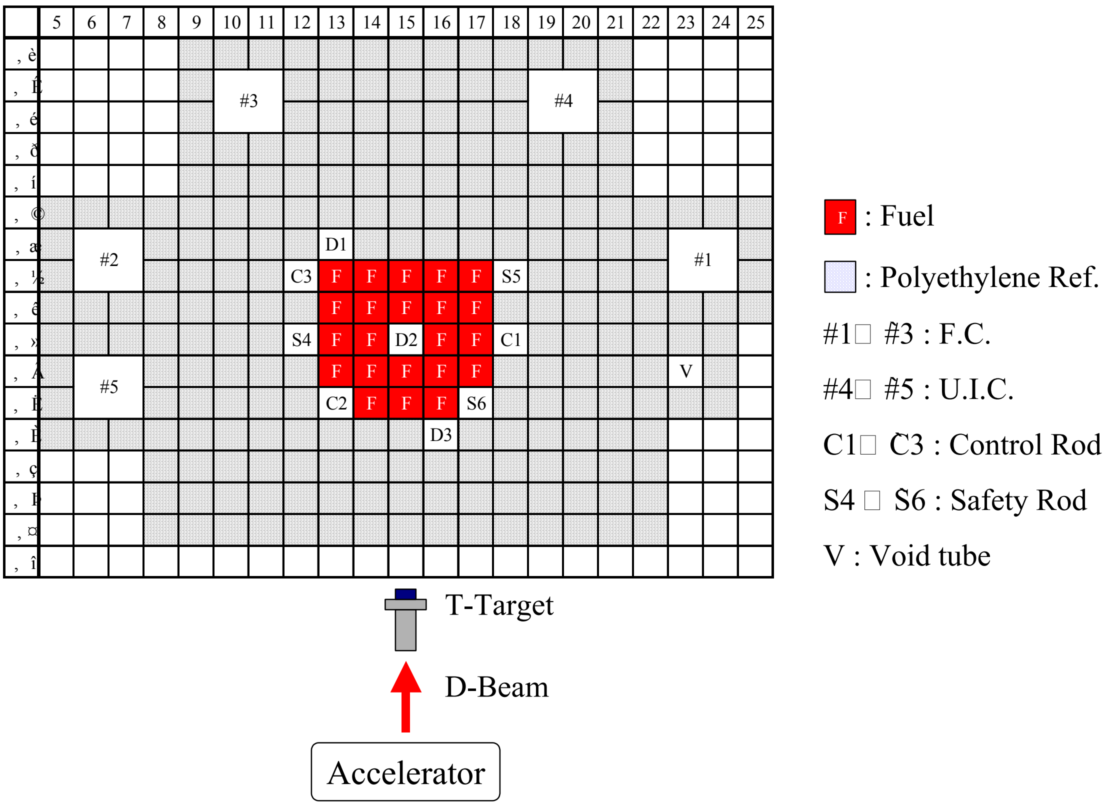

Figure 3. Schematic plan view of experimental system at KUCA.

Typical experimental results of Y-value in the system with the subcriticality of 1.7$ are depicted in Figure 4. The experimental result showed the same temporal behavior as ones shown in Figure 1 and 2. The prompt neutron decay constants evaluated by the least-square fitting procedure are 445 ± 4 s^{-1} and 604 ± 21 s^{-1} for the subcriticality of 1.7$ and 3.6$, respectively. On the other hand, the corresponding values were 391 s^{-1} and 764 s^{-1}, which were evaluated by applying the conventional pulsed neutron method to the same time series data of neutron counts. The value obtained by V-to-M method for 1.7$ subcritical core agrees with one by pulsed neutron method with its accuracy of 13%. This result is not sufficient from the viewpoint of rigorous subcriticality estimation, but is valuable as a demonstration of the validity of new formula.

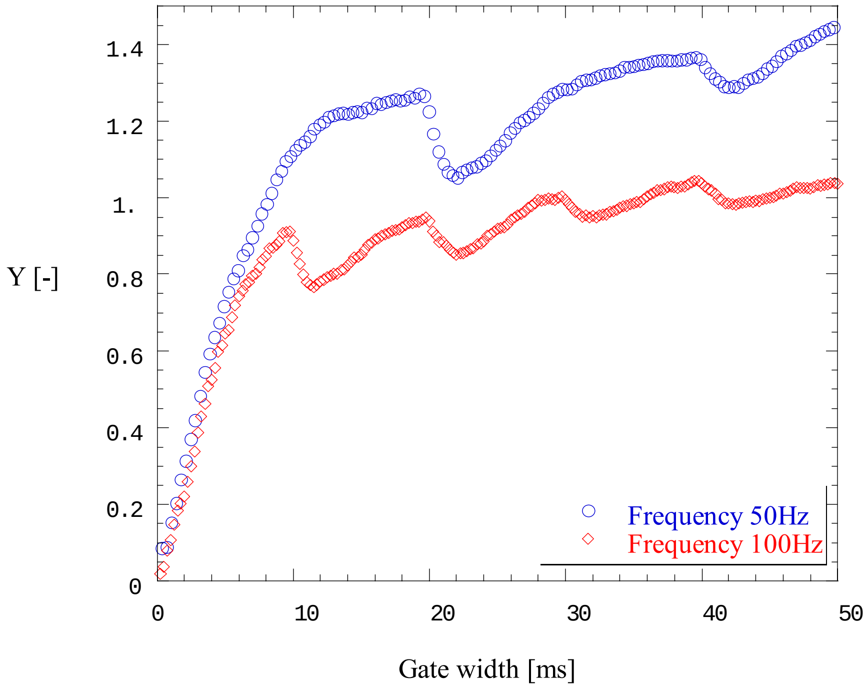

Figure 4. Experimental results of Y-value in 1.7$ subcritical core.

4. CONCLUSION

Through the present studies, it is concluded that the new V-to-M method for the quasi-steady state can be derived, and the prompt neutron decay constant can be estimated by using it. Hence, in spite of the ADS operation mode, the V-to-M method is applicable to estimate the subcriticality of the ADS system by virtue of the prompt neutron decay constant. To accomplish this method as the subcrticality monitor, further study from the quantitative viewpoint is necessary in the future.

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