ABSOLUTE MEASUREMENT OF THE SUBCRITICALITY BY USING THE THIRD ORDER MOMENT OF THE NUMBER OF NEUTRONS DETECTED

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ABSTRACT

Absolute measurement of the subcriticality was carried out by using a neutron correlation technique that uses a third order moment of the number of neutrons detected in the experiments carried out at the Kyoto University Critical Assembly. Although the experimental errors were not so small, it was confirmed that the present technique could roughly give absolute values of the subcriticality of down to 4.5 %$\Delta k/k$ when locations of the neutron detector and the extraneous neutron source were carefully chosen.

1. INTRODUCTION

Measurement of the negative reactivity (subcriticality) has been one of the most important subjects in the field of reactor physics, because it is generally required to achieve a safer and more economic operation of facilities where the nuclear fuel is dealt with. A large number of studies have been performed so far to measure the subcriticality by using a few experimental techniques; the area ratio, the neutron correlation techniques, and so forth.

The area ratio technique where short bursts of neutrons are repeatedly injected into a subcritical system is regarded to be one of the most reliable techniques for measuring the subcriticality. However, this technique requires a pulsed neutron generator or a pulse tube equipment, so that it is not applicable to facilities where such equipment is not installed. On the other hand, the neutron correlation techniques, such as the variance-to-mean (Feynman-α) and Rossi-α ones, require no special equipments except for a neutron detector and an extraneous neutron source, since the subcriticality is determined by measuring the neutron counts from the detector located in the subcritical system where the neutron flux is maintained by the neutron source. However, unfortunately, the absolute measurement of the subcriticality cannot be carried out without knowing the prompt neutron decay constant at the critical state α_c.

Recently, we have intended to examine an experimental technique that was firstly proposed by Furuhashi and Izumi[1] through a series of experiments using the Kyoto University Critical Assembly (KUCA) at the Kyoto University Research Reactor Institute. The technique is based on the variance-to-mean one, but can determine the absolute value of the subcriticality without α_c. In the present paper, the first results of the examination where locations of the neutron detector and the extraneous neutron source were carefully chosen to avoid some difficulties due to the spatial effect will be reported.

2. THEORY

2.1 Y- AND X-VALUES

Let us assume that time series data of the number of neutrons detected with respect to a counting gate time (gate width) T are repeatedly acquired by using a multi channel scaler (MCS) that has successive N counting channels. In the variance-to-mean technique, $Y_i(T)$ that is the correlation index Y for the i-th MCS's run (i = 1, 2, …, M) is calculated as follows:

$$Y_i(T) \equiv \frac{v_i(T)}{\mu_i(T)} - 1, \tag{1}$$

where $\mu_i(T)$ and $v_i(T)$ are the mean and the variance of the number of neutrons detected for the i-th run, and when the neutron counts stored in the j-th counting channel for the i-th run is denoted by $Z_{i,j}(T)$, these are defined as,

$$\mu_i(T) \equiv \frac{1}{N}\sum_{j=1}^{N} Z_{i,j}(T), \quad v_i(T) \equiv \frac{1}{N}\sum_{j=1}^{N} Z_{i,j}^2(T) - \left\{\frac{1}{N}\sum_{j=1}^{N} Z_{i,j}(T)\right\}^2. \tag{2}$$

Using the one-point reactor approximation that neglects delayed neutrons, the expected value of Y can be derived as follows[2]:

$$\langle Y(T)\rangle \equiv \frac{\langle v(T)\rangle}{\langle \mu(T)\rangle} - 1 = Y_{\text{sat}} f_1(\alpha T, N) - \frac{1}{N}, \tag{3}$$

$$\langle \mu(T)\rangle = \frac{\lambda_{\text{d}}\langle q\rangle S}{\alpha} T, \quad \langle v(T)\rangle = Y_{\text{sat}}\langle \mu(T)\rangle f_1(\alpha T, N) + \frac{N-1}{N}\langle \mu(T)\rangle, \tag{4}$$

$$\alpha \equiv -\frac{\rho}{\Lambda}, \quad \rho \equiv \frac{\langle \nu\rangle \lambda_{\text{f}} - (\lambda_{\text{c}} + \lambda_{\text{f}} + \lambda_{\text{d}})}{\langle \nu\rangle \lambda_{\text{f}}}, \quad \Lambda \equiv \frac{1}{\langle \nu\rangle \lambda_{\text{f}}}, \tag{5}$$

$$Y_{\text{sat}} \equiv \frac{\lambda_{\text{d}}\lambda_{\text{f}}\langle \nu(\nu-1)\rangle}{\alpha^2}(1+\delta_2), \quad \delta_n \equiv \frac{\langle \nu\rangle\langle q(q-1)\cdots(q-n+1)\rangle}{\langle q\rangle\langle \nu(\nu-1)\cdots(\nu-n+1)\rangle}(-\rho), \tag{6}$$

$$f_1(x,y) \equiv 1 - \frac{1-\mathrm{e}^{-x}}{x} - \frac{1}{y}\left(1 - \frac{1-\mathrm{e}^{-xy}}{xy}\right), \tag{7}$$

where brackets means the ensemble average over i. The nomenclatures used in Eqs. (3) ~ (7) are identical to those defined in Ref. 3 except for δ_n that is a generalized one.

Here, we would like to consider the definition of Y, again. One can easily find from Eq. (4) that when the definition of Y is modified as,

$$Y_i^*(T) \equiv \frac{v_i(T)}{\mu_i(T)} - \frac{N-1}{N}, \tag{8}$$

the theoretical expression of the expected value can be slightly simplified as follows:

$$\langle Y^*(T)\rangle = Y_{\text{sat}} f_1(\alpha T, N). \tag{9}$$

In the present study, this modified index Y^* will be used.

In 1968, Furuhashi and Izumi proposed a higher order version of the variance-to-mean technique by introducing a third order correlation index X that is defined as,

$$X_i(T) \equiv \frac{s_i(T)}{\mu_i(T)} - 3\frac{v_i(T)}{\mu_i(T)} + 2, \tag{10}$$

$$s_i(T) \equiv \frac{1}{N}\sum_{j=1}^{N} Z_{i,j}^3(T) - 3\left\{\frac{1}{N}\sum_{j=1}^{N} Z_{i,j}^2(T)\right\}\left\{\frac{1}{N}\sum_{j=1}^{N} Z_{i,j}(T)\right\} + 2\left\{\frac{1}{N}\sum_{j=1}^{N} Z_{i,j}(T)\right\}^3, \tag{11}$$

and derived the theoretical expression of its expected value[1]. In the present study, the X value was also modified for simplicity as,

$$X_i^*(T) \equiv \frac{s_i(T)}{\mu_i(T)} - \frac{3(N-2)}{N}\frac{v_i(T)}{\mu_i(T)} + \frac{2(N-1)(N-2)}{N^2}, \tag{12}$$

and the theoretical expression of the expected value of X^* was newly derived as follows:

$$\left\langle X^*(T)\right\rangle = X_{2,\mathrm{sat}} f_2(\alpha T, N) + X_{3,\mathrm{sat}} f_3(\alpha T, N), \tag{13}$$

$$X_{2,\mathrm{sat}} \equiv \frac{3\lambda_\mathrm{d}^2\lambda_\mathrm{f}^2\left\langle \nu(\nu-1)\right\rangle^2}{\alpha^4}(1+\delta_2), \quad X_{3,\mathrm{sat}} \equiv \frac{\lambda_\mathrm{d}^2\lambda_\mathrm{f}\left\langle \nu(\nu-1)(\nu-2)\right\rangle}{\alpha^3}(1+\delta_3), \tag{14}$$

$$\begin{aligned} f_2(x,y) \equiv\ & 1 + \mathrm{e}^{-x} - \frac{2-2\mathrm{e}^{-x}}{x} - \frac{1}{y}\left\{3 + \mathrm{e}^{-x} - \frac{4-4\mathrm{e}^{-x}}{x} - \frac{2-2\mathrm{e}^{-xy}}{xy}\left(1 - \frac{x\,\mathrm{e}^{-x}}{1-\mathrm{e}^{-x}}\right)\right\} \\ & + \frac{2}{y^2}\left(1 + \mathrm{e}^{-xy} - \frac{2-2\mathrm{e}^{-xy}}{xy}\right), \end{aligned} \tag{15}$$

$$\begin{aligned} f_3(x,y) \equiv\ & 1 - \frac{3+\mathrm{e}^{-2x}-4\mathrm{e}^{-x}}{2x} - \frac{1}{y}\left[3 - \frac{3-3\mathrm{e}^{-x}}{x} - \frac{\left(1-\mathrm{e}^{-x}\right)\left(1-\mathrm{e}^{-xy}\right)}{xy}\left\{1 + \frac{1+\mathrm{e}^{-xy}}{2\left(1+\mathrm{e}^{-x}\right)}\right\}\right] \\ & + \frac{2}{y^2}\left(1 - \frac{3+\mathrm{e}^{-2xy}-4\mathrm{e}^{-xy}}{2xy}\right). \end{aligned} \tag{16}$$

2.2 SUBCRITICALITY MEASUREMENT

From Eq. (14), it can be found that when a correlated neutron source such as the ^{252}Cf is employed, the next equation,

$$(-\rho) = \frac{\langle q\rangle\langle \nu(\nu-1)\rangle}{\langle \nu\rangle\langle q(q-1)\rangle}\left(\frac{3Y_{\mathrm{sat}}^2}{X_{2,\mathrm{sat}}} - 1\right), \tag{17}$$

can be utilized for measuring the subcriticality. However, one have to note here that Eq. (17) becomes meaningless when a Poisson neutron source such as the Am-Be is employed, because q is always equal to unity[3].

In order to determine $X_{2,\mathrm{sat}}$, one will be able to utilize Eq. (13), however, the next approximated formula,

$$\left\langle X^{*}(T)\right\rangle \simeq X_{2,\mathrm{sat}} f_{2}(\alpha T, N), \tag{18}$$

seems to be more useful when the subcriticality is not so large. Let us consider the ratio of $X_{3,\mathrm{sat}}$ to $X_{2,\mathrm{sat}}$ in order to verify this approximated formula:

$$\frac{X_{3,\mathrm{sat}}}{X_{2,\mathrm{sat}}} = \frac{\langle\nu\rangle\langle\nu(\nu-1)(\nu-2)\rangle}{3\langle\nu(\nu-1)\rangle^{2}} \frac{(1+\delta_{3})}{(1+\delta_{2})}(-\rho). \tag{19}$$

Figure 1 shows the ratio of $X_{3,\mathrm{sat}}$ to $X_{2,\mathrm{sat}}$ as a function of the subcriticality when a ^{252}Cf neutron source is employed[4]. One can easily understand that when the subcriticality is not so large, the $X_{3,\mathrm{sat}}$ is much smaller than $X_{2,\mathrm{sat}}$ and hence the second term of Eq. (13) becomes negligible.

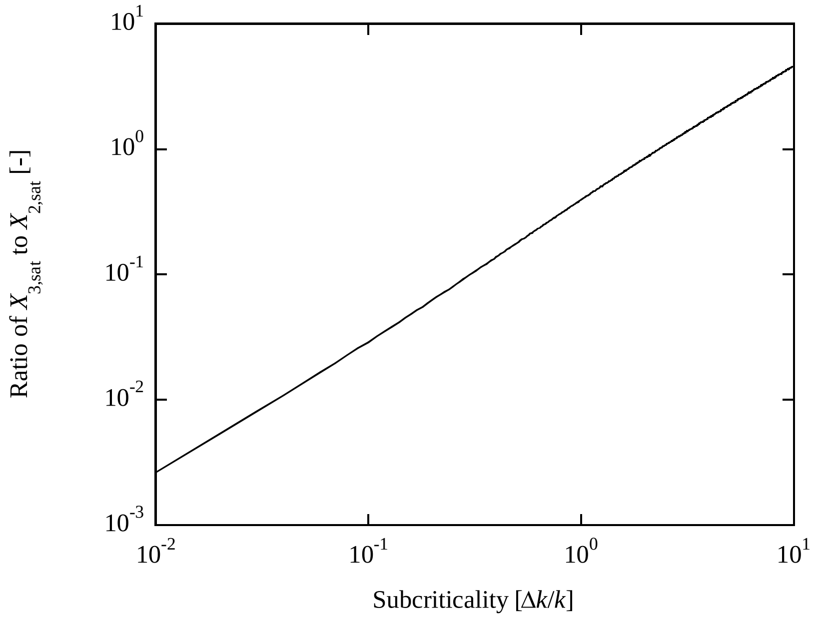

Figure 1. Ratio of $X_{3,\mathrm{sat}}$ to $X_{2,\mathrm{sat}}$ when a ^{252}Cf neutron source is employed.

3. EXPERIMENTAL

The KUCA has three independent cores named A, B and C. Among them, the A-core where polyethylene moderators and reflectors are mainly used was employed. The core system in the present study was built of twenty-one ordinary fuel elements, a special one that includes a ^{252}Cf neutron source, and the detector element as shown in Fig. 2. The fuel element was assembled by inserting 36 unit cells between upper and lower polyethylene reflectors as shown in Fig. 3. The unit cell of the fuel element consisted of a coupon type highly enriched uranium-aluminum fuel plate (50.8 mm square, 1.6 mm thickness, 93% enrichment) and two kinds of polyethylene plates (50.8 mm square, 3.2 and 6.4 mm thicknesses). In order to assemble the special fuel element that was located at (f-5), the polyethylene plate of 3.2 mm thickness in the 19th unit cell from the lower side was replaced with that of the same thickness in which a ^{252}Cf neutron source was inlaid. The height of thus constructed core region, the lower and the upper polyethylene reflectors were approximately 40, 50 and 56 cm, respectively. A couple of BF_3 detectors (#1 and #2) of 12.8 mm diameters were inserted into the detector element (see Fig. 4), and the element was located at (e-5) to avoid the spatial effect as possible. The detectors were connected to conventional nuclear instrumentation sets; pre- and main amplifiers, and timing single channel analyzers (TSCAs). The output pulses from the TSCAs were fed into a 4-input type MCS that has 8,192 successive counting channels.

In the present study, by arranging stroke patterns of control and safety rods that were placed adjacent to the fuel elements, time series data of successive neutron counts were acquired by using the MCS at three subcritical states.

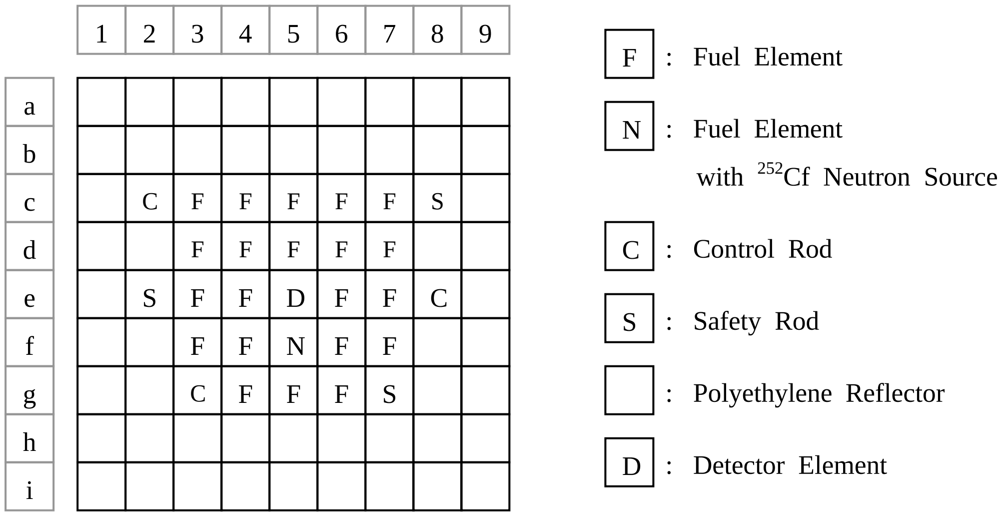

Figure 2. Horizontal Cross Section of the Core Configuration (KUCA, A-Core).

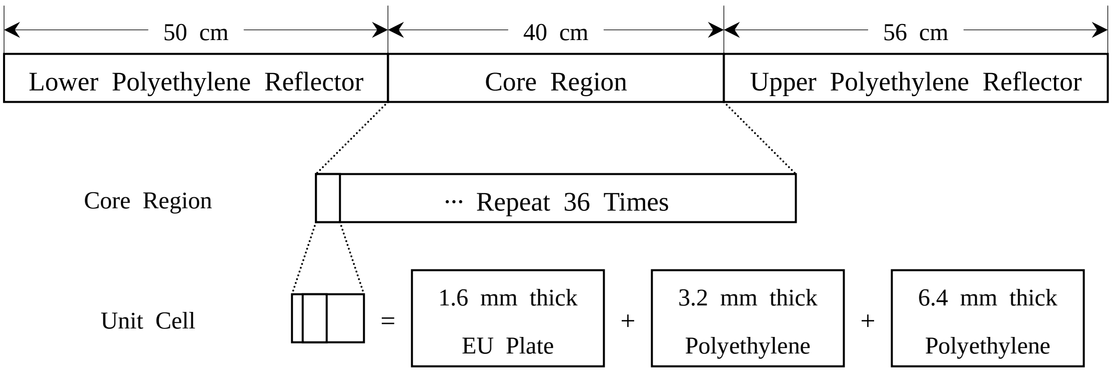

Figure 3. Composition of the Fuel Element.

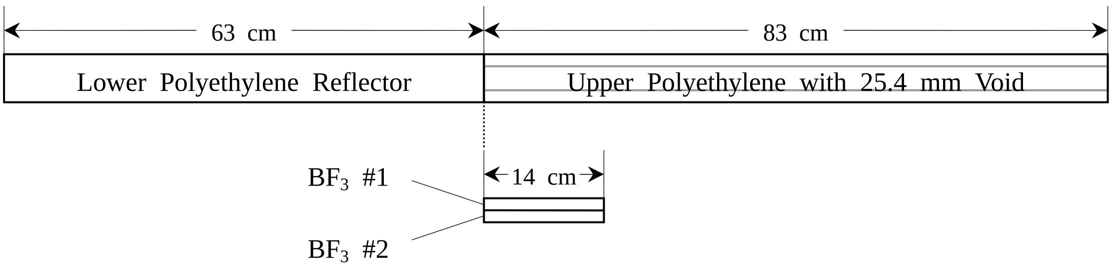

Figure 4. Composition of the Detector Element.

4. RESULTS AND DISCUSSION

Figures 5, 6 and 7 are the measured results of the Y^* and the X^* values with respect to the neutron detector #1. These figures include the fitted curves for the Y^* values that were obtained by the least square method with Eq. (9). On the other hand, in the present experiment, since $X_{2,sat}$ is expected to be much greater than $X_{3,sat}$, the fitted curves for the X^* values were obtained by using an approximated formula, *i.e*. Eq. (18). Thus measured subcriticalities are listed in Table I. In order to calculate the subcriticalities from Eq. (17), the next value,

$$\frac{\langle q\rangle\langle\nu(\nu-1)\rangle}{\langle\nu\rangle\langle q(q-1)\rangle}\simeq 0.598, \qquad (20)$$

was employed[4]. As a reference, the measured results by the area ratio technique are also listed in Table I. From these results, it is found that some results by the present technique considerably differ from those by the area ratio one and the experimental errors are not small. However, it appears that the present technique can roughly give absolute values of the subcriticality of down to 4.5 $\%\Delta k/k$ when locations of the neutron detector and the extraneous neutron source are carefully chosen.

Table I. Measured Subcriticalities by the Area Ratio and the Present Techniques.

	Detector	Area-Ratio Technique ($\%\Delta k/k$)	Present Technique ($\%\Delta k/k$)
Subcritical I	#1	1.25 ± 0.01	1.46 ± 1.06
	#2	1.26 ± 0.01	0.98 ± 1.01
Subcritical II	#1	2.77 ± 0.05	*
	#2	2.67 ± 0.04	6.40 ± 2.61
Subcritical III	#1	4.49 ± 0.05	3.98 ± 3.91
	#2	4.48 ± 0.07	4.87 ± 5.30

(* the subcriticality could not be determined since it became negative)

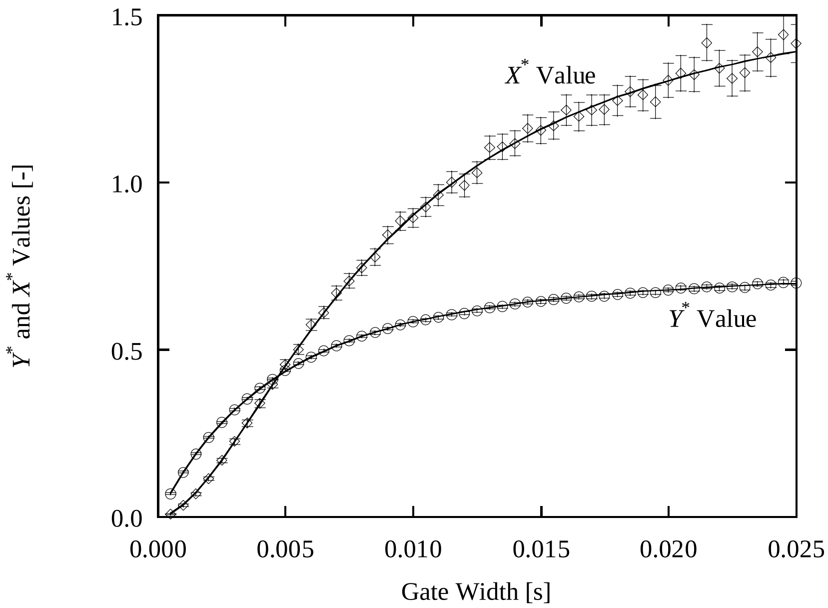

Figure 5. Example Result of Y^* and X^* Values (Subcritical I, Detector #1).

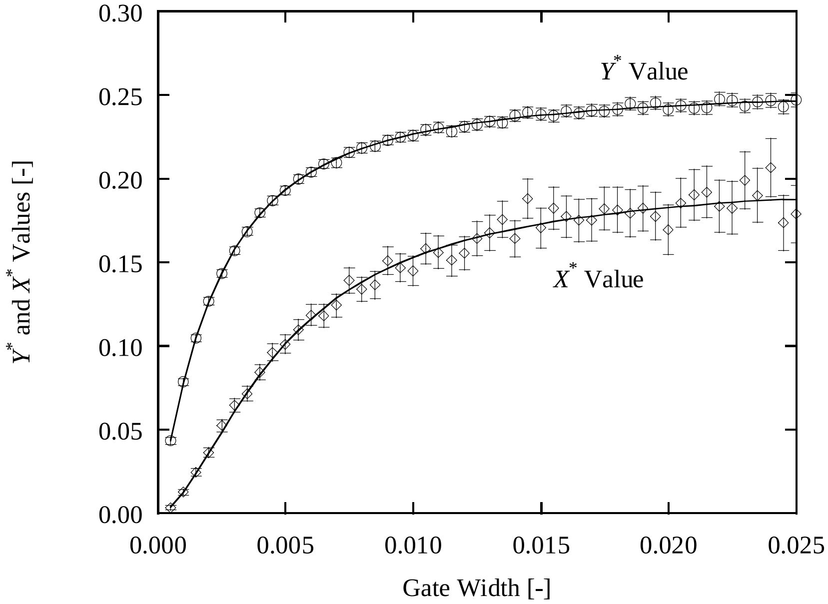

Figure 6. Example Result of *Y** and *X** Values (Subcritical II, Detector #1).

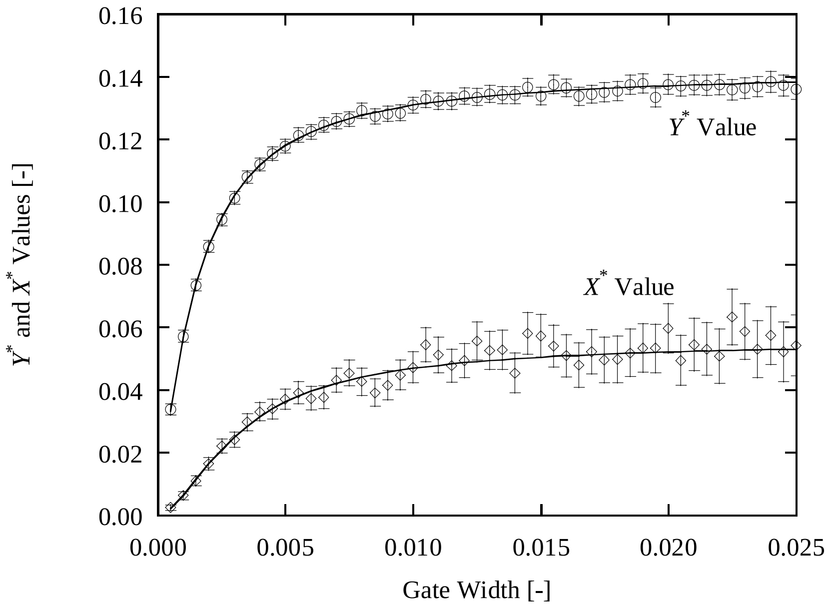

Figure 7. Example Result of *Y** and *X** Values (Subcritical III, Detector #1).

5. CONCLUSIONS

Through the experiments carried out at the KUCA, absolute measurement of the subcriticality was performed by using the third order moment of the number of neutrons detected. Although some results by the present technique somewhat differed from those by the area ratio technique and the experimental errors were not so small, it was confirmed that the present technique could roughly give absolute values of the subcriticality of down to 4.5 %$\Delta k/k$ when locations of the neutron detector and the extraneous neutron source were carefully chosen.

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