A Simple and Flexible Coarse Mesh Method for Reactor Core Simulator

Tetsuo Matsumura, Takanori Kameyama and Yasushi Nauchi
Central Research Institute of Electric Power Industry (CRIEPI),
11-1, Iwado Kita 2-Chome, Komae-Shi, Tokyo 201-8511 Japan
Tel: +81-3-3480-2111 Fax: +81-3-3480-2493
E-mail: matsu-t@criepi.denken.or.jp

ABSTRACT

Simple and flexible two and three-dimensional diffusion and transport coarse mesh methods are presented for high burn-up UO_2 and MOX fuel core simulation. Present coarse mesh methods use coefficients of simple 5-points (2D) or 7-points (3D) type differential equation produced by normal finite differential method (FDM) or FDM-like transport calculation with fine meshes. The present methods have advantages on treatment of heterogeneity within fuel assemblies. To reduce total calculation time, eigenvalue (outer) iterations are carried out in coarse mesh calculation. IAEA 3D benchmark calculation and UO_2 / MOX fuel assemblies calculation show that the present methods are about 10 times faster than conventional methods in calculation times.

1. INTRODUCTION

Due to burn-up extension of UO_2 fuel and usage of MOX (Mixed Oxide) fuels in a LWR core, complexity of reactor core simulation comes to be increased. This complexity causes necessity of three-dimensional analyses and transport effect consideration in LWR core simulation. As these calculations need enormous computing resource, various computing methods are proposed and developed to reduce computing time and memory sizes (unknown values) [1-3]. Coarse mesh methods (node methods) achieved success in reactor core simulation. However there is room for improvement on discontinuous factors of neutron fluxes between neighboring fuel assemblies of high burn-up UO_2 fuels and MOX fuels.

Reactor core simulation by three-dimensional finite differential method (FDM) with fine (fuel pin cell level) meshes treats heterogeneity within fuel assemblies, and needs no discontinuous factors between fuel assemblies. We have developed a three-dimensional diffusion coarse mesh method, which uses three-dimensional FDM calculation with fine meshes for inner (neutron flux distributions) calculation, and 7-points type differential coarse mesh equation for outer (eigenvalue and neutron source distributions) calculation. The present method need no discontinuous factor between fuel assemblies, and acquire remarkable reduction of computing time compared with normal FDM with fine meshes.

Transport calculation needs more computing resource than diffusion calculation extremely. Recently various acceleration methods with diffusion type calculations are presented such as diffusion synthetic acceleration (DSA) schemes [2, 3]. We had presented “Solution of Sn-type transport Equation by finite differential method”, which uses FDM-like 5-points type

differential equation for a two-dimensional Sn-type transport equation [4]. In the paper, we present more simple FDM-like method of Sn-type transport calculation using 5-points type differential equation. Additionally the present FDM-like transport calculation method can be easily combined with previous coarse mesh method.

To evaluate performance of these coarse mesh methods, numerical calculations are carried out for IAEA three-dimensional diffusion benchmark problem [5] and UO_2 / MOX fuel assemblies transport calculation problem.

2. COARSE MESH METHOD FOR THREE DIMENSIONAL DIFFUSION CALCULATION

Normal three-dimensional finite differential diffusion method (FDM) uses 7-points type differential equation:

$$C_0\ \phi_{i,j,k} = C_1\ \phi_{i-1,j,k} + C_2\ \phi_{i+1,j,k} + C_3\ \phi_{i,j-1,k} + C_4\ \phi_{i,j+1,k} + C_5\ \phi_{i,j,k-1} + C_6\ \phi_{i,j,k+1} + S_{i,j,k} \quad (1)$$

where, C_0 to C_6 are differential coefficients for diffusion term for mesh (i,j,k), $\phi_{i,j,k}$ and $S_{i,j,k}$ are total flux and source of mesh (i,j,k).

Coarse mesh (I,J,K) and coarse mesh averaged neutron flux ($\Phi_{I,J,K}$) are defined as Fig. 1. Neutron leakage ($J_{I,J,K}$) of coarse mesh $\Phi_{I,J,K}$ to coarse mesh $\Phi_{I-1,J,K}$ can be calculated with differences between neutron fluxes of fine meshes as:

$$J_{I,J,K} = \underset{j,k\ \in\ J,K}{\text{Sum}}\ 2\cdot D_{i,j,k} D_{i-1,j,k} / (D_{i,j,k}\Delta X_{i-1,j,k} + D_{i-1,j,k}\Delta X_{i,j,k})\cdot \Delta Y_{i,j,k}\cdot \Delta Z_{i,j,k}\cdot(\phi_{i-1,j,k} - \phi_{i,j,k}) \quad (2)$$

Where, $D_{i,j,k}$, $\Delta X_{i,j,k}$, $\Delta Y_{i,j,k}$ and $\Delta Z_{i,j,k}$ are diffusion coefficient and X, Y, Z-direction mesh widths of fine mesh (i,j,k). Summation covers X-direction boundary fine meshes within the coarse mesh (I,J,K).

Therefore differential coefficients for coarse mesh can be calculated such as:

$$C_1(I,J,K) = \frac{\underset{j,k\ \in\ J,K}{\text{Sum}}\ 2\cdot D_{i,j,k} D_{i-1,j,k} / (D_{i,j,k}\Delta X_{i-1,j,k} + D_{i-1,j,k}\Delta X_{i,j,k})\cdot \Delta Y_{i,j,k}\cdot \Delta Z_{i,j,k}\cdot \phi_{i-1,j,k}}{\Phi_{I-1,J,K}} \quad (3)$$

and

$$\mathbf{C}_0(I,J,K) = \mathbf{C}_1(I+1,J,K) + \mathbf{C}_2(I-1,J,K) + \mathbf{C}_3(I,J+1,K) + \mathbf{C}_4(I,J-1,K) + \mathbf{C}_5(I,J,K+1) + \mathbf{C}_6(I,J,K-1) + \underset{i,j,k \in I,J,K}{\mathrm{Sum}} (\Sigma a_{i,j,k} + \Sigma r_{i,j,k}) \cdot \Delta X_{i,j,k} \cdot \Delta Y_{i,j,k} \cdot \Delta Z_{i,j,k} \cdot \phi_{i,j,k} \Big/ \Phi_{I,J,K} \quad (4)$$

where, $\Sigma a_{i,j,k}$ and $\Sigma r_{i,j,k}$ are absorption and removal cross sections of mesh (i,j,k). Coarse mesh 7-points type differential equation is:

$$\mathbf{C}_0(I,J,K)\,\Phi_{I,J,K} = \mathbf{C}_1(I,J,K)\,\Phi_{I-1,J,K} + \mathbf{C}_2(I,J,K)\,\Phi_{I+1,J,K} + \mathbf{C}_3(I,J,K)\,\Phi_{I,J-1,K} + \mathbf{C}_4(I,J,K)\,\Phi_{I,J+1,K} + \mathbf{C}_5(I,J,K)\,\Phi_{I,J,K-1} + \mathbf{C}_6(I,J,K)\,\Phi_{I,J,K+1} + \mathbf{S}_{I,J,K} \quad (5)$$

where, $\mathbf{S}_{I,J,K}$ is source of coarse mesh (I,J,K) calculated as:

$$\mathbf{S}_{I,J,K} = \underset{i,j,k \in I,J,K}{\mathrm{Sum}} S_{i,j,k} \quad (6)$$

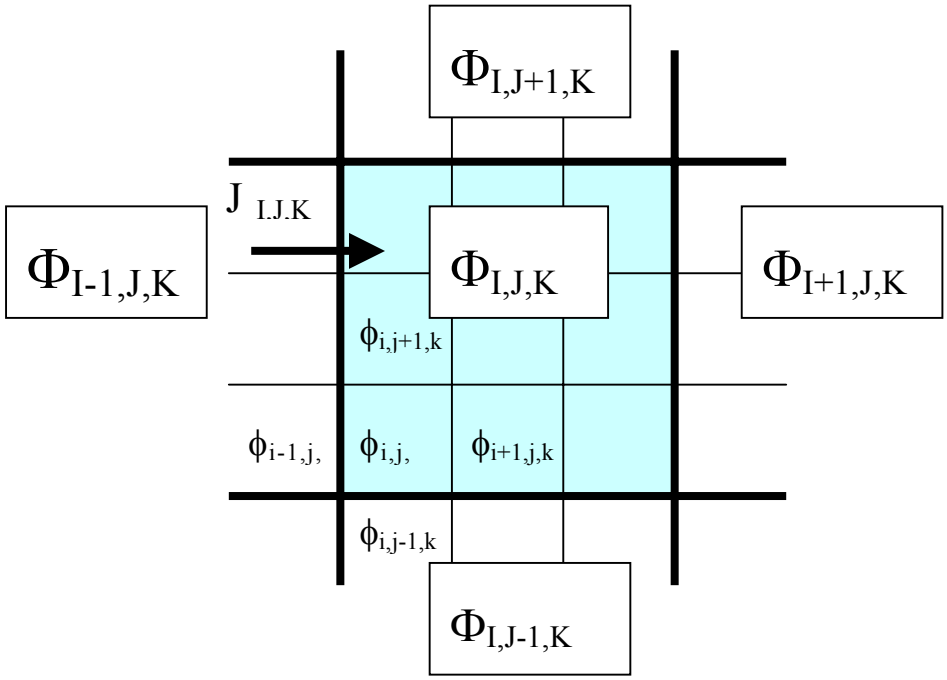

Fig. 1 Neutron fluxes of fine meshes and coarse meshes.

The present coarse mesh calculation scheme starts with a normal FDM calculation of fine mesh flux distributions (see Fig. 2). Differential coefficients for coarse mesh are produced with the equation (3) and (4). Source distribution and eigenvalue are calculated with coarse mesh calculation of the 7-points type differential equation (5). Coarse mesh calculation loop is necessary to achieve consistency between fine mesh flux distributions and coarse mesh flux distributions. Termination of coarse mesh calculation loop is decided with convergence of coarse mesh source distributions or differential coefficients of coarse meshes.

The present coarse mesh method has similarity to usual coarse mesh acceleration methods of diffusion calculation. However the present coarse mesh method is able to omit outer iteration of fine mesh FDM and reduce total calculation time.

The present method has advantages on:

- Treatment of heterogeneity within a coarse mesh such as fuel assemblies.
- Flexibility of calculation geometries such as xyz, rθz or rθψ coordinate systems.
- Possibility of triangle meshes.

by using suitable FDM for inner calculation. Therefore the present coarse mesh method has simplicity and flexibility.

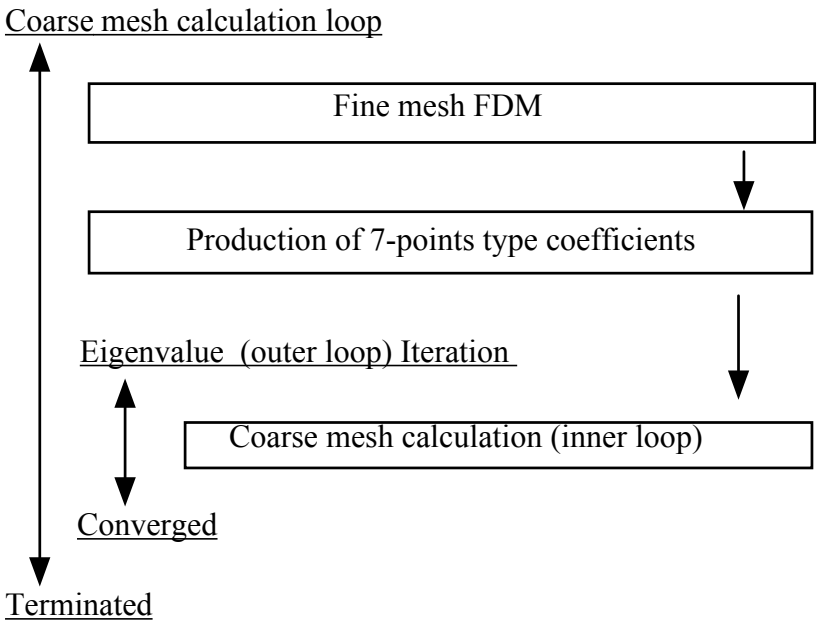

Fig. 2 Calculation scheme of the present coarse mesh method.

3. FDM-LIKE TRANSPORT CALCULATION METHOD

A transport calculation considers angular neutron flux. Outgoing boundary angular flux of a mesh $\{\psi_{1,m}(x)\}$ is calculated with boundary income angular neutron flux $\{\psi_{0,m}(x)\}$ and transmission probability (T_m) of the two dimensional mesh and angle (m) as [4,6]:

$$\psi_{1,m}(x) = \psi_{0,m}(x) \cdot T_m \qquad (7)$$

$T_m = \{1 - \exp(-\Sigma t \Delta / \mu_m)\} / (\Sigma t \Delta / \mu_m)$ for area A and C (triangle)

$T_m = \exp(-\Sigma t \Delta / \mu_m)$ for area B (trapezoid)

where, Σt is total cross section, μ_m is direction cosines of angle m. Boundary angular fluxes are defined for x-axis boundary (x) and y-axis boundary (y) (see Fig. 3).

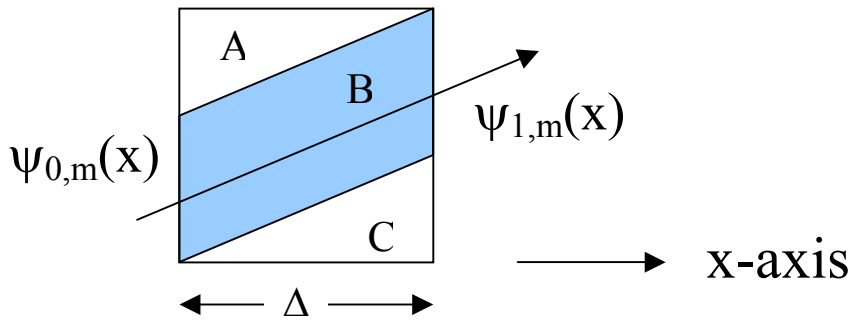

Fig. 3 Areas of a mesh for transmission calculations.

Income neutron current ($J^+_{i,j}$) of mesh (i,j) can be presented as:

$$J^+_{i,j} = \underset{m \in M}{\text{Sum}} \; \psi_{i,j,m}(X) \; w_m \; \Delta Y_{i,j} \qquad (8)$$

where, w_m is weight of angle m, $\Delta Y_{i,j,}$ is Y-direction mesh width of mesh (i,j). Summation covers X^+-direction of M (see Fig. 4).

X and y boundary angular fluxes $\{\psi_m(x)$ and $\psi_m(y)\}$ are calculated with normal sweep method for every angle of m. 5-points type differential coefficients for the mesh (i,j) can be presented with $\psi_m(x)$ and $\psi_m(y)$ such as:

$$C_1(i,j) = \mathrm{Sum}_{m \in M} \psi_{i,j,m}(x) \; w_m \; \Delta Y_{i,j} \; / \phi_{i-1,j} \tag{9}$$

and

$$C_0(i,j) = C_1(i+1,j)+C_2(i-1,J)+C_3(i,j+1)+C_4(i,j-1) + (\Sigma t_{i,j} - \Sigma s_{i,j}) \cdot \Delta X_{i,j,} \cdot \Delta Y_{i,j} \tag{10}$$

where, $\Sigma s_{i,j}$ is scattering cross sections of mesh (i,j).

Total neutron flux distributions (ϕ) are calculated with these 5-points type differential coefficients. However additional angular flux convergence loop is necessary to achieve consistency between angular neutron flux distributions (ψ) and the total neutron fluxes (ϕ). In the paper, we simplify calculation method of differential coefficients with boundary angular fluxes such as Equation (9) and (10). We present two-dimensional FDM-like transport calculation methods here. The present method can be extended to three dimensional transport calculation method with 7-points type differential coefficients.

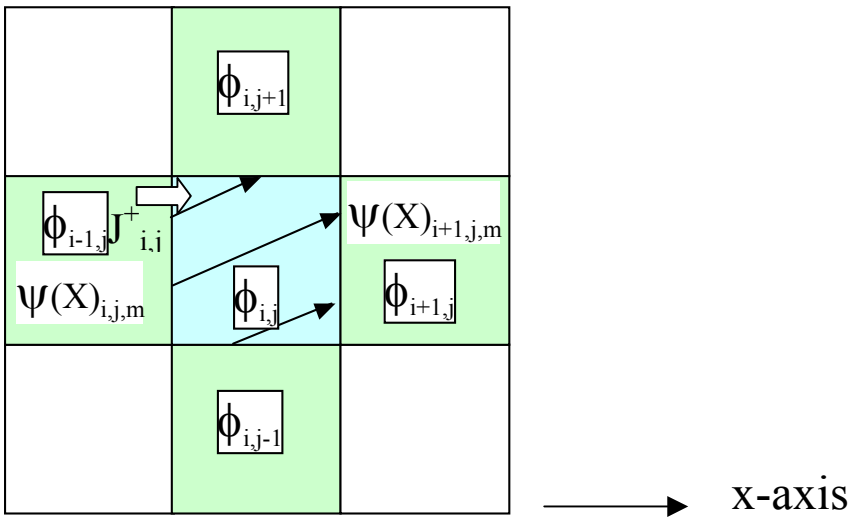

Fig. 4 Neutron fluxes and angular fluxes of meshes.

4. COURCE MESH METHOD FOR TWO DIMENSIONAL TRANSPORT CALCULATION

The present coarse mesh method can be easily applied for the FDM-like transport calculation method. However differential coefficients for coarse mesh are calculated with Equation (9) for boundary of coarse mesh (I,J). Fig. 5 shows the present coarse mesh transport method (CMT) calculation scheme that is similar to that of the diffusion coarse mesh methods (Fig.

2). The FDM-like (fine meshes) transport calculation method needs additional angular flux convergence loop to achieve consistency between angular neutron flux distributions and the total neutron fluxes. However "coarse mesh transport calculation loop" plays both of convergence loop of "angular flux and (fine meshes) total fluxes" and "fine meshes total fluxes and coarse mesh total fluxes". The present CMT method has advantages on:

- Flexibility of angle numbers for neutron angular flux calculation
- Better convergence performance by differential type equations
- Possibility of arbitrary square meshes [6] and triangle meshes

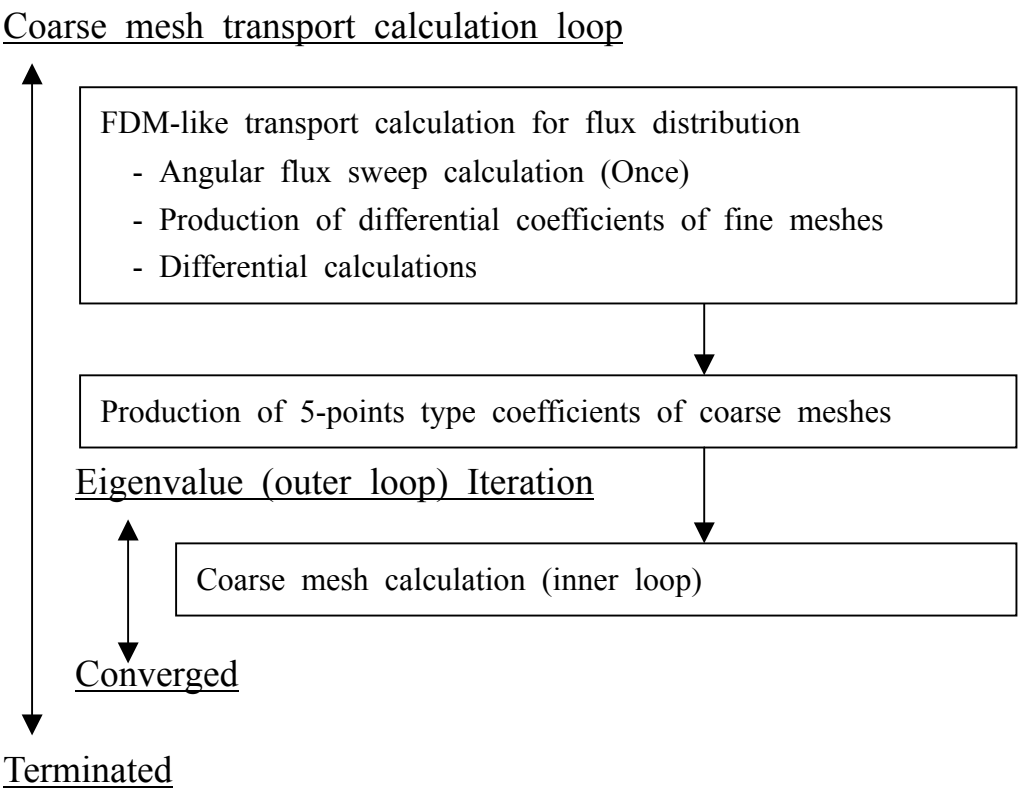

Fig. 5 Calculation scheme of the present 2D coarse mesh transport (CMT) method.

5. TEST CALCULATIONS OF THE COARSE MESH METHODS

5.1 Coarse mesh method for 3D diffusion calculation

IAEA 3D benchmark calculation is one of standard benchmark problems [5]. Geometry and cross sections of the benchmark problem are shown in Fig. 6 and Table 1.

Table 1 Two-group constants for IAEA 3D benchmark problem

Region	D_1	D_2	Σ_r	$\Sigma_{a,1}$	$\Sigma_{a,2}$	$\upsilon\Sigma_{f,2}$
Fuel Type-1	1.5	0.4	0.02	0.01	0.08	0.135
Fuel Type-2	1.5	0.4	0.02	0.01	0.085	0.135
Fuel 2+CR	1.5	0.4	0.02	0.01	0.13	0.135
Reflector	2.0	0.3	0.04	0.0	0.01	0.0
Refl.+CR	2.0	0.3	0.04	0.0	0.055	0.0

Cross sections : cm^{-1}, Diffusion coefficients: cm.

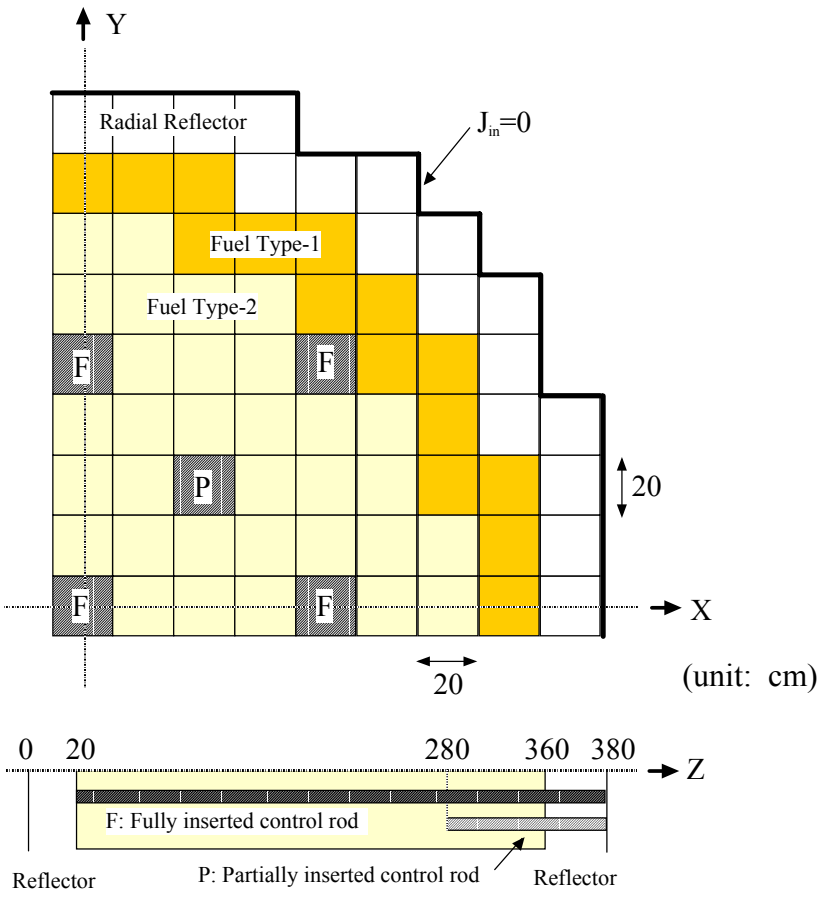

Fig. 6 Geometries of the IAEA 3D benchmark problem.

Fig. 7 shows relation between CPU time and assembly power errors with conventional FDM and the present method in the IAEA 3D benchmark problem. Assembly power errors are relative difference from the values of reference power distribution shown in the literature [5]. The present coarse mesh method show remarkable reduction of CPU time compared with the fine mesh conventional FDM on the IAEA 3D benchmark problem. The present coarse mesh method shows 10 times faster than fine FDM of the same mesh sizes. However the present method shows a little larger values of assembly power errors than the fine FDM of the same mesh sizes, due to difference between convergence criteria of both methods. Eigenvalues of the present coarse mesh calculation show good agreement with the reference value (1.02903)

within 0.1% δk / k. These calculations are carried out with personal computer and COMPAC visual FORTRAN 6.5 compiler.

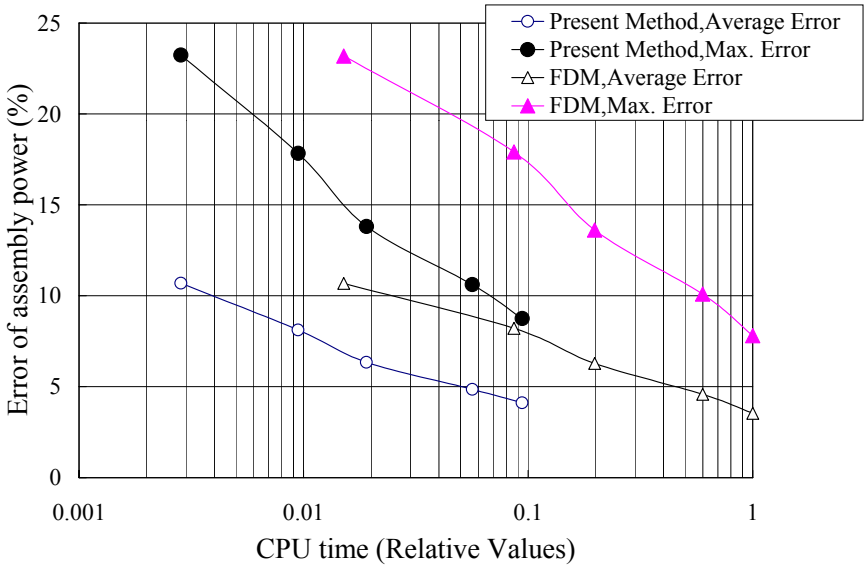

Fig. 7 Relation between CPU time and assembly power errors by conventional FDM and the present method in the IAEA 3D-benchmark problem.

5.2 Coarse mesh method for 2D transport calculation

The present methods have advantages on treatment of heterogeneity within fuel assemblies. We set up a UO_2 / MOX fuel assemblies transport calculation problem shown in Fig. 8, which has heterogeneity of Gd_2O_3 doped UO_2 fuel rods in UO_2 fuel assemblies and Pu enrichment distribution in a MOX fuel assembly. Additionally there are heterogeneity between UO_2 and MOX fuel assemblies. We used geometries and characteristic data of UO_2 / MOX fuel assemblies defined in a literature [7]. Macro cross sections of each fuel rod cell are listed in Table 2, which are obtained with fuel rod cell calculations using JENDL-3.2 [8] and the ANISN code [9].

Table 2 Two group constants for a UO_2 / MOX fuel assemblies transport calculation problem

(Unit: cm^{-1})

Region	$\Sigma_{t,1}$	$\Sigma_{t,2}$	$\Sigma_{s,1}$	$\Sigma_{s,2}$	Σ_r	$\upsilon\Sigma_{f,1}$	$\upsilon\Sigma_{f,2}$
UO_2 Fuel Rod (6.5 w%)	0.2784	0.8925	0.2529	0.7575	0.0132	0.0114	0.2552
UO_2-Gd_2O_3 Fuel Rod	0.2761	1.3594	0.2464	0.8230	0.0130	0.0071	0.0363
Guide Tube, Inst. Tube*	0.2349	1.0433	0.2107	1.0364	0.0235	-	-
MOX Fuel Rod (12.2 w%)	0.2799	1.2064	0.2526	0.7406	0.0079	0.0210	0.7107
MOX Fuel Rod (9.2 w%)	0.2807	1.1456	0.2539	0.7374	0.0088	0.0176	0.6285
MOX Fuel Rod (4.8 w%)	0.2816	1.0306	0.2558	0.7439	0.0106	0.0118	0.4481
Guide Tube, Inst. Tube**	0.2255	0.9280	0.2107	0.9221	0.0142	-	-

* Water region of UO_2 fuel assemblies ** Water region of MOX fuel assembly

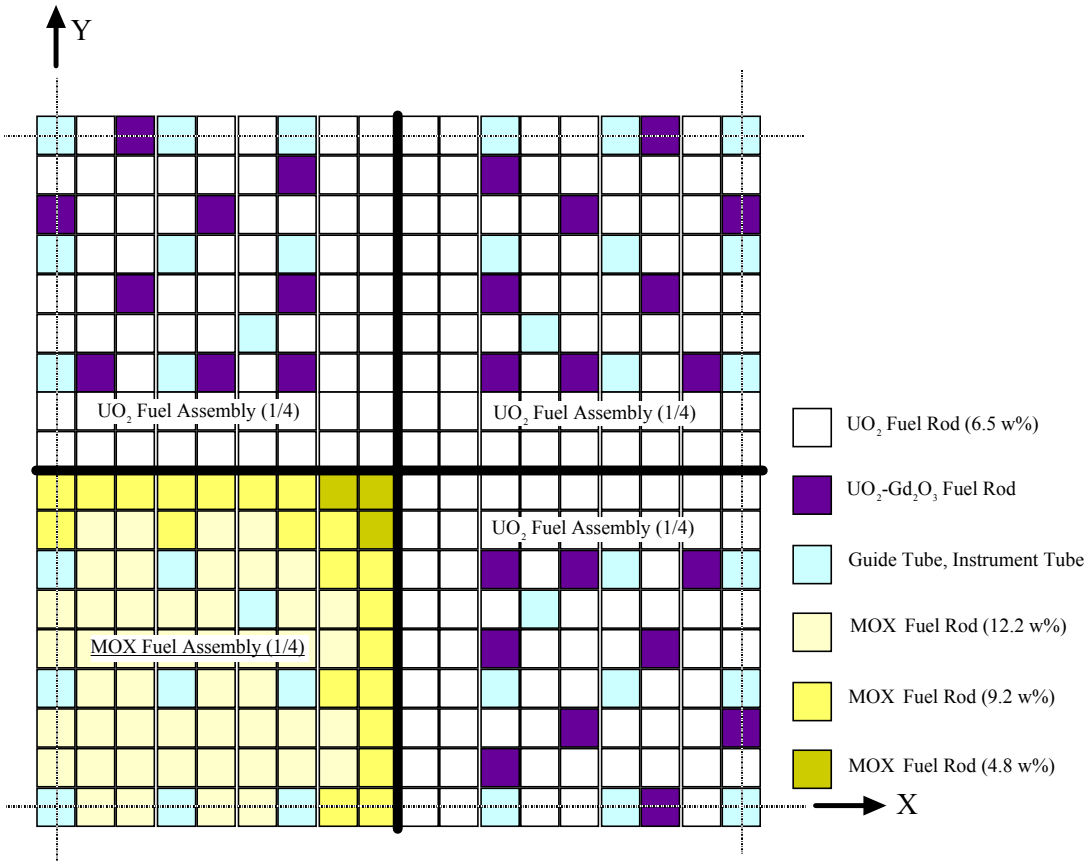

Fig. 8 Geometries and main characteristic data of a UO_2 / MOX fuel assemblies transport calculation problem.

Calculations are carried out with the FDM-like transport calculation method, the present CMT method and FLEXBURN code [6]. Fuel rod cells are divided into some fine meshes. Coarse meshes are set as fuel assemblies, that is, 2 x 2 meshes in the CMT calculations. Fig. 9 is an example of power distribution of the UO_2 / MOX fuel assemblies, which shows heterogeneity within and between UO_2 / MOX fuel assemblies. Differences of the calculated values are small between the FDM-like transport calculation method and the CMT method. Differences of eigenvalues are less than 0.03 % $\delta k / k$ for the same fine mesh size (Fig. 10). The maximum and averaged differences of power distributions are less than 1 % and 0.2 % between both calculation methods respectively. The FDM-like transport calculation method shows very good agreement with FLEXBURN, which is a well-verified transport calculation code [7]. Fig. 9 shows that the MOX fuel assembly has larger relative power than the UO_2 fuel assemblies. This result is not general in commercial LWRs, because enrichments are not adjusted in the calculation problem.

The CMT method shows remarkable reduction of CPU time (Fig. 11). It is about 10 times faster than FDM-like transport calculation methods of the same fine meshes.

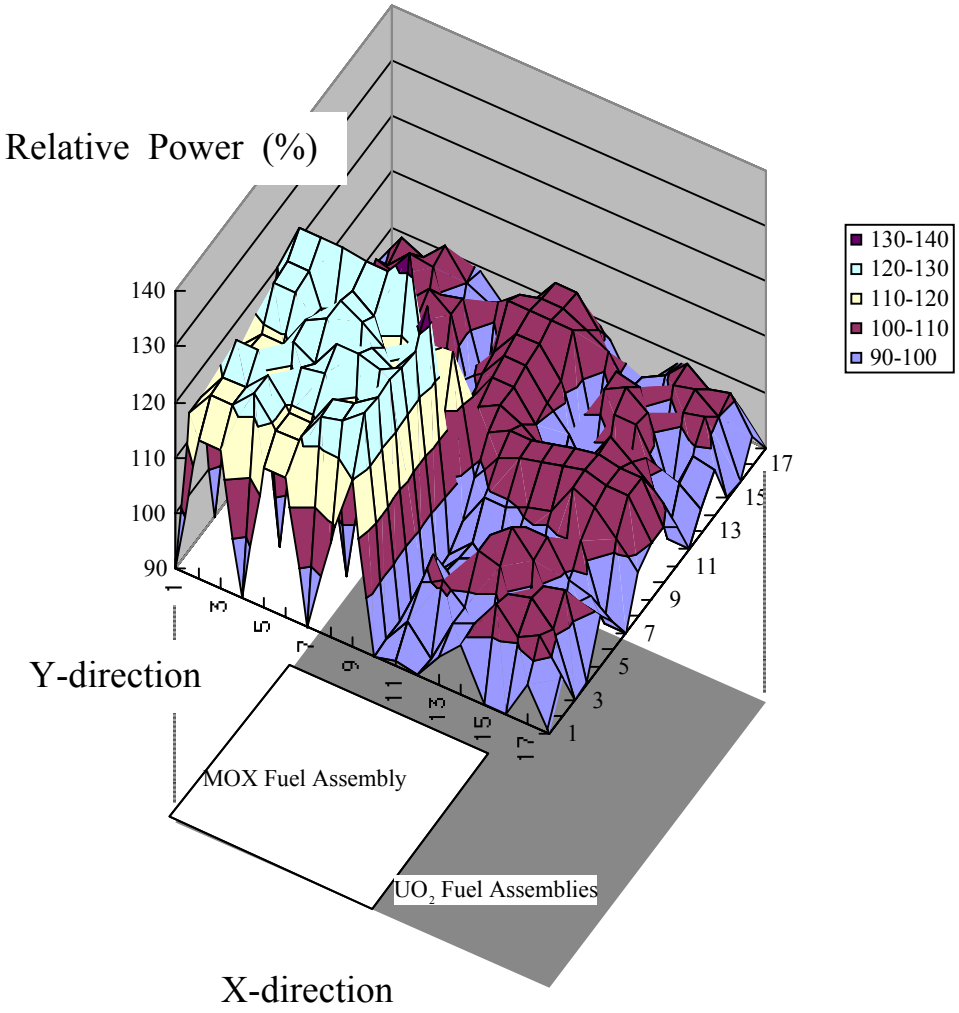

Fig. 9 An example of power distribution of the UO_2 / MOX fuel assemblies. (FDM-like transport calculation, 8x8 meshes / fuel rod cell)

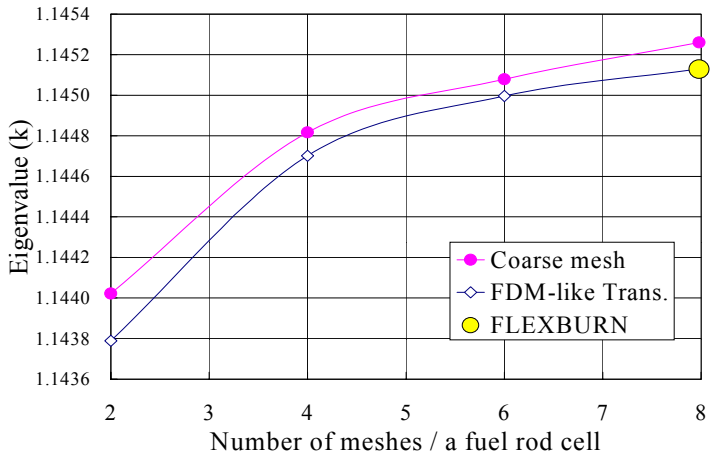

Fig. 10 Eigenvalues of the UO_2 / MOX fuel assemblies calculated with the FDM-like transport method and the CMT method.

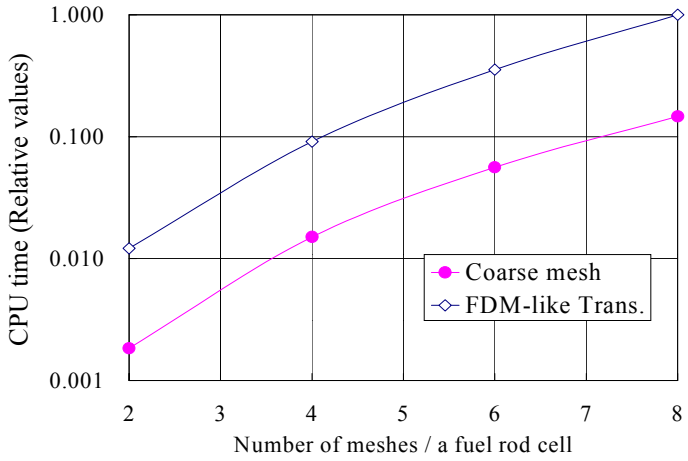

Fig. 11 Relative CPU times of the FDM-like transport method and the CMT method for the UO_2 / MOX fuel assemblies calculations.

CONCLUSIONS

We developed simple and flexible two and three-dimensional diffusion and transport coarse mesh methods for high burn-up UO_2 and MOX fuel core simulation. Calculation equations and scheme are presented for three-dimensional diffusion coarse mesh methods, which uses coarse mesh coefficients of 7-points type differential equation produced by normal finite differential method (FDM) with fine meshes. To reduce total calculation time, eigenvalue (outer) iterations are carried out in coarse mesh calculation. The present method has advantages on treatment of heterogeneity within a coarse mesh such as fuel assemblies, flexibility of calculation geometries (including triangle meshes) by using simple FDM for inner calculation. IAEA 3D benchmark calculation show remarkable (7-8 times) reduction in CPU times compared with fine mesh FDM.

Transport calculation needs more computing resource than diffusion calculation. We had presented FDM-like differential equation method, which uses Sn-type transmission probabilities of meshes and 5-points type differential equation. In the present paper, we simplify the FDM-like transport calculation method. The present FDM-like transport method has advantages on flexibility of angle numbers for angular flux calculation, better convergence performance by differential type equation and possibility of arbitrary square mesh and triangle meshes. Additionally the FDM-like transport calculation method can be easily applied to the present coarse mesh method.

The present coarse mesh transport (CMT) methods have advantages on treatment of heterogeneity within fuel assemblies. We set up a UO_2 / MOX fuel assemblies transport calculation problem, which has heterogeneity of Gd_2O_3 doped UO_2 fuel rods in UO_2 fuel assemblies and Pu enrichment distribution in a MOX fuel assembly. Additionally there are heterogeneity between UO_2 and MOX fuel assemblies. The CMT method shows remarkable (about 10 times) reduction of CPU time compared with the FDM-like transport method for the same fine mesh size. Therefore, the CMT method has possibility of applying full core calculation. As the CMT methods have simplicity and flexibility, extension to three dimensional transport calculation is planned in the future.

REFERENCES

1. M. Nakagawa, T. Takeda, et al., “New or Improved Computational Methods and Advanced Reactor Design”, *J. AESJ*, **Vol. 39**, No. 1 (1997) [in Japanese]

2. M. L. Adams and W. R. Martin, “Diffusion Synthetic Acceleration of Discontinuous Finite Element Transport Iterations”, *NSE*, **111**, 145-167 (1992)

3. J. E. Morel, J.E. Dendy, Jr., and T.A. Wareing, “Diffusion-Accelerated Solution of the Two-Dimensional Sn Equations with Bilinear-Discontinuous Differencing”, *NSE*, **115**, 304-319 (1993)

4. T, Matsumura and T. Kameyama, ”Solution of Sn-type Transport Equation by Total Flux Finite Difference Method”, *Proceeding of Physor 96,* A-15, Mito, Japan (1996)

5. “Argonne Code Center: Benchmark Problem Book”, ANL-7416, Supple. 2 (1977)

6. T. Kameyama, T. Matsumura, and M. Sasaki, "The FLEXBURN Neutron Transport Code Developed by the Sn Method with Transmission Probabilities in Arbitrary Square Meshes for Light Water Reactor Fuel Assemblies," *NSE*, **123**, 86-95 (1996).

7. “Proposal and Analysis of the Benchmark Problem Suite for Reactor Physics Study of LWR Next Generation Fuels”, *JAERI-Research* 2001-046 (2001) [in Japanese]

8. T. Nakagawa, et al., “Japanese Evaluated Nuclear Data Library Version 3 Revision-2: JENDL-3.2”, *J, Nucl. Sci. Technol*., **Vol. 32**, pp. 1259-1271 (1995)

9. W.W.Engle, Jr., "The Users Manual for ANISN: A One-Dimensional Discrete Ordinated Transport Code with Anisotropic Scattering," K-1694, Oak Ridge Gaseous Diffusion Plant (1967).