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LOADING PATTERN SEARCH BY BRANCHING AND BOUNDING BATCH PATTERNS ENUMERATED UNDER CONSTRAINTS

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ABSTRACT

A new loading pattern (LP) search method, Branching and Bounding Batch Patterns Enumerated under Constraints (B3PEC), has been developed. B3PEC is a deterministic rather than stochastic search method, using the technique of constrained enumeration and the technique of Branch and Bound (B&B) mixed integer linear programming to perform comprehensive searches. The method resembles the approach that core design engineers are used to think of LP search, but with a systematic process based on sound theory. Batch Loading Patterns (BLPs) containing coarse batches of "identical" fuel assemblies are exhaustively enumerated with user specified loading constraints. Via enumeration, a user can explore potentially viable strategies of loading and can pre-determine, before any spatial calculation, the appropriate strategy and the associated problem size. As the search progresses, the coarse batches get divided into finer batches using B&B mixed integer linear programming with additional design constraints, and end up with real LPs containing distinct individual assemblies. This process allows all LPs under the constraints to be covered and each LP is visited only once. Only a production core design code (e.g. the Westinghouse ANC code) is used for the spatial analysis of BLPs and LPs. No additional neutron diffusion module is needed. This

eliminates inconsistency between the LP search code and the design code. The total number of resulting LPs depends on the total number of initial BLPs and the levels of BLP refinement. In any case, the total explored domain in the B3PEC method is immensely larger than that of other LP search methods. This process yields multiple solutions with distinctly different features, which can be directly used for core designs. The B3PEC method has been applied to a new loading pattern search code system, LP-FUN (Loading Patterns For User's Need).

1. INTRODUCTION

In general, a loading pattern search method contains three basic steps: (1) generating patterns by shuffling assemblies, (2) performing the spatial flux and power distribution calculation, and (3) evaluating an objective function and ranking patterns for acceptance or rejection. Although the most time consuming part in LP search is spatial calculation, shuffling is actually the most crucial part as it controls the search domain and hence the effectiveness of a search method. The relatively straightforward part is evaluation and ranking which are neither theoretically involved nor computationally time consuming. However, so far most LP search methods in literature focus on the methods for spatial analysis and objective function evaluation, without providing a "theory" or "methodology" for shuffling itself. For example, both simulated annealing and genetic algorithms address objective function evaluations for decision making. The generalized perturbation theory method addresses how to perform the spatial analysis. In the absence of a systematic method for generating and pre-assessing LPs, shuffling can only be sampled heuristically and stochastically. By the time a shuffle is evaluated by an LP search code to be detrimental, it is too late, as computing time has already been wasted in the spatial analysis of the LP resulting from the shuffle. Furthermore, LP search methods based on heuristic and stochastic shuffling do not provide a sense of how good a search solution really is nor to what extent the problem has been solved, as one can not "visualize" the explored domain. There is no information about where the domain is, how big it is, how much has been explored relative to the total solution space, whether the same solutions have been repeatedly visited or if the search is trapped in a local minimum. Consequently, solutions from stochastic LP search methods are not reproducible and can not be objectively benchmarked.

The work reported here shifts the emphasis and focus of LP search to step 1 in the above discussion: the methodology of assembly shuffling and pattern generation. Core designers know that the most important decision in LP search is where the feed assemblies go and where highly burnt assemblies go. This kind of "batch" loading pattern grossly determines the major characteristics of an LP. In practical designs, engineers work with these coarse BLPs towards refinement. The difficulty here is that a designer does not know how many there are of these BLPs and it is a tedious job to track down and refine a BLP manually. There are too many possible BLPs and each can sprout into an extremely large number of refinement paths. Consequently, a designer can only manually investigate a very limited number of BLP and its derived LPs. However, recently Si [1] successfully demonstrated an efficient recursive enumeration, which generates BLPs deterministically and systematically. This enumeration technique makes it practically feasible to exhaustively generate all BLPs under loading constraints. As for the tracking of each BLP down the path of refinement into real LP, it involves shuffling of assemblies only in the same batch, which do not have grossly different reactivity. The refinement of BLPs can therefore be treated as a linear programming problem, which can be very efficiently solved with the B&B mixed integer linear programming method. Using the enumeration technique and the B&B mixed integer linear programming, we have developed the B3PEC search method and applied it to the new LP search code LP-FUN. Section 2 discusses the method and process of B3PEC, and Section 3 presents two examples of LP-FUN application and discusses its practicality. Section 4 concludes the paper.

2. THE B3PEC METHOD AND PROCESS

The B3PEC search process starts with batch loading patterns. A batch is defined as a collection of “identical” assemblies. Initially the patterns contain coarse batches, such that all the allowable BLPs that satisfy user-specified batch loading constraints are exhaustively enumerated. As the search progresses, the coarse batches get branched into finer batches. After one or two levels of branching, further branching can go directly to distinct individual assemblies providing real LPs. The branching process is carried out with the B&B mixed integer linear programming method with additional design constraints accommodated.

The problem size and the domain to be searched are first explored and assessed with the enumeration algorithm, Batch Pattern Enumeration under Constraints (BPEC). This algorithm employs only logic and integer operations and can efficiently generate each and all of the allowable BLPs under a variety of flexible position constraints. Forbidden regions of core positions can be defined for each batch where loading is not allowed. Any number of forced domains can be defined for each batch, where a minimum and/or maximum number of assemblies have to be loaded. Constraints to ban various types of clustering among assemblies of specified batch or batches can be imposed in defined regions. The above regions and domains are arbitrary and can overlap, and each one can be of a topologically disconnected shape. Using BPEC, a user can pre-determine, before any spatial calculation starts, how big a search problem is to be solved under what kind of loading constraints. A user can use BPEC repeatedly until he is satisfied with the chosen strategy on loading constraints and the associated problem size.

When the “identical” assemblies in a batch are split into distinct and finer batches, the shuffling of the distinct assemblies within the original batch to generate the daughter patterns is an integer permutation problem, which leads to an enormously large number of daughter patterns. However, if this discrete integer permutation problem is first converted to a continuous real variable problem by imagining that the distinct assemblies could be arbitrarily taken apart and reassembled as a mixture, then the continuous variable problem can be readily solved with powerful mathematical tools. (For example, by differentiating a function of a continuous variable, one finds directly the minimum. While in the discrete variable case, the function has to be repeatedly evaluated at all discrete points to obtain its minimum.) The best continuous solution always “bounds” (at least as good as) the best possible discrete integer solution. If the best continuous solution is not acceptable, then there is no point of analyzing and tracking all the discrete permutations from this BLP. Hence this whole BLP “branch” can be pruned. Only those surviving BLPs will be tracked further. This process of refining BLPs is carried out with B&B mixed integer linear programming. This BLP refinement process repeats until real LPs with individually distinct assemblies appear. Kim and Kim [2] also used B&B mixed integer programming in their work. But they did not take the approach of enumerating all BLPs, nor generalizing the idea of B&B to multi-level refinement of BLPs.

The B3PEC method can be implemented by using only a production core design code for the spatial analysis of BLPs and LPs without any additional neutron diffusion module. This eliminates the inconsistency between the LP search code and the design code. In LP-FUN the Westinghouse core design code ANC is used, with its 3D to 2D collapsed model, as the spatial analysis tool. At each BLP refinement level, for each BLP, ANC 2D depletion is repeatedly performed to directly generate a sensitivity matrix (S-matrix) with respect to both burnable absorber adjustment and burn-up exposure variation. The S-matrix is input to a mixed integer linear programming algorithm to perform B&B optimization, on burnable absorber assignment and in-batch shuffling, for the current level BLP to reach the next level of refined BLPs. The mixed integer linear program can accommodate any design constraint that can be expressed as a linear combination of assembly average power, assembly peak power and the core critical boron concentration at all the depletion steps. The levels of refinement on BLPs that are needed before reaching real LPs depend on the available fuel inventory for the problem. Typically only one or two levels of batch refinement are needed before going directly to the individually distinct assemblies. The total number of resulting LPs depends on the total number of initial BLPs and the levels of BLP refinement. In any case, the total explored domain in the B3PEC method is immensely larger than that of other LP search methods. The resulting solutions are multiple, each with distinctly different features, which can be directly used by ANC for core designs.

3. EXAMPLES OF LP-FUN APPLICATION

3.1 AN EXAMPLE OF ENUMERATION AND SCOPING

Here we give a simple example of how the enumeration module BPEC can be used to assess the problem size and properly define the scope of search. Checkerboard BLPs, with three batches of equal size, are considered in this example for Westinghouse type 2, 3, and 4-loop cores. No two assemblies in the same batch are allowed to be adjacent unless one of the two is on the periphery. As shown in Table I, when no additional constraints are imposed the number of solutions are very large, about ten thousand for 2-loop core, a hundred thousand for a 3-loop core and two million for a 4-loop core. If the additional constraint is imposed that batch 1 (feeds) can not be in a specified central region, then the solutions drop dramatically to about two thousand, four thousand and six thousand respectively. If another constraint is added that only feeds can be on the periphery (out/in pattern), then there are only 4, 31 and 41 solutions. All these patterns are explicitly generated by BPEC very fast except for the case of two million solutions.

Table I. Number of 3-Batch, Non-peripheral Checkerboard BLPs

Additional Constraint	2-Loop	3-Loop	4-Loop
None	12,342	127,932	1,995,472
No Feeds in Central Region	1,798	3,692	5,649
And Only Feeds on Periphery	4	31	41

To show how effectively BPEC can be used to define the scope of search, the 5649 4-loop BLPs in Table I, where no feeds are allowed in a central region, are analyzed dimensionally at beginning of cycle (BOC). Figure 1 displays the distribution of core peak power for all 5649 BLPs. Obviously many of the patterns are not good at all. To improve the search space, we now impose an additional new constraint, yet relax one old constraint. We divide the core into an inboard region and an outboard region. The additional constraint is that in the inboard region, only the low reactivity twice burnt fuel (batch 3 in this case) can be adjacent to the feeds to balance out the high reactivity. This constraint should get rid of many high peak patterns and reduce the number of solutions. Then we remove the constraint that the once burnt fuel (batch 2 in this case) can not cluster with itself. This is reasonable, because although one would like to smear out and mix assemblies of high and low reactivity, there is no reason why the medium reactivity assemblies have to be spread out as well. The relaxing of this constraint should increase the number of acceptable solutions. With these two changes, the total number of solutions change from 5649 to 4583. As we can see in Figure 2, these solutions are now much better than the ones in Figure 1. All high peak power solutions are removed and there are many more solutions with low peak power. This demonstrates that using very simple and generic physical argument, one can effectively assess and define the search scope.

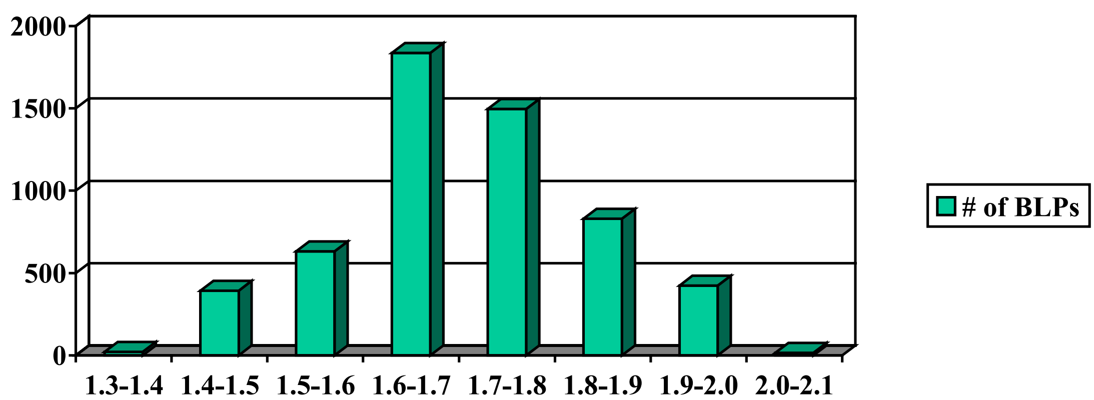

Figure 1. Distribution of Core Peak Power at BOC for the 5649 BLPs in Table I.

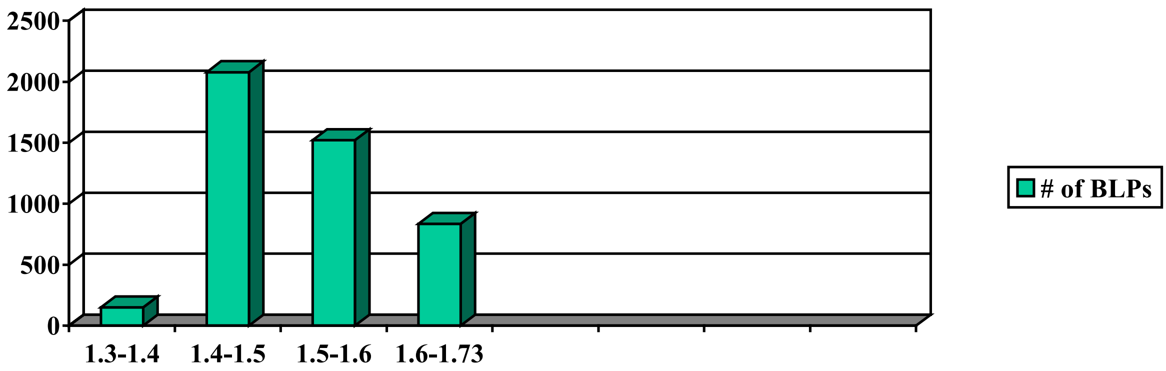

Figure 2. Distribution of Core Peak Power at BOC for the 4583 BLPs With Improved Contraints.

3.2 AN EXAMPLE OF COMPLETE SEARCH

This example demonstrates the application of LP-FUN to a complete LP search process for a 4-loop reload core. This core has 68 feed assemblies using gadolinium as burnable absorber, 69 once burnt fuel assemblies and 56 twice burnt fuel assemblies. The design is for a partial low leakage core. We start with BPEC enumeration using 4 coarse batches. They are then split to 6 batches before going directly to the distinct individual assemblies.

For this problem, after running BPEC a few times the following position constraints are chosen for enumerating the initial 4-batch BLPs. The feeds must be loaded in about 60% of the peripheral region (any combination of peripheral locations) and forbidden in the 9 locations at the core center. No two of feeds can be adjacent to each other anywhere unless one of them is on the periphery. Gadolinium can not be loaded in peripheral feeds. The core is divided in an inboard region and an outboard region. A once burnt assembly is not allowed to be adjacent to any feed or another once burnt assembly in the inboard region, so that only twice burnt assemblies and feeds can be adjacent in the inboard region to smear out the reactivity distribution there. In the outboard region, some degree of clustering among burnt assemblies is allowed, such as a cluster up to three assemblies. Under these constraints, BPEC enumerated 4175 BLPs. Each one of the 4175 BLPs can sprout into an enormous number of LP when the assemblies in batches are made distinct and permutated in location.

The 4175 BLPs are analyzed with ANC at BOC. The peak pin power and the critical boron concentration at BOC for all the BLPs are displayed in Figure 3. It is obvious that many of them are bad patterns not worthy of pursuing. The patterns with peak pin power greater than 1.7 are thrown away. There are 613 patterns left with peak power less than 1.7, and they are analyzed with batch refinement. The four batches are now split into six batches, with each of the two burnt fuel batches split into two batches. For each of the 613 4-batch BLPs, an S-matrix is generated at BOC in order to calculate power distribution and core reactivity response to splitting a burnt batch into two batches. The S-matrix is input to a B&B mixed integer linear programming code to search for the 6-batch BLP that has the lowest power peak among all the 6-batch BLPs that can sprout from a given 4-batch BLP. After this batch refinement only 136 6-batch BLPs survive, with all the other BLPs violating the imposed peak pin power limit of 1.45. Figure 4 shows how the 613 4-batch BLPs evolve to the 136 6-batch BLPs. Also shown in Figure 4 is the comparison of the S-matrix prediction versus the ANC prediction at BOC for the 136 6-batch BLPs. The two predictions agree very well except for a relatively constant bias in boron concentration.

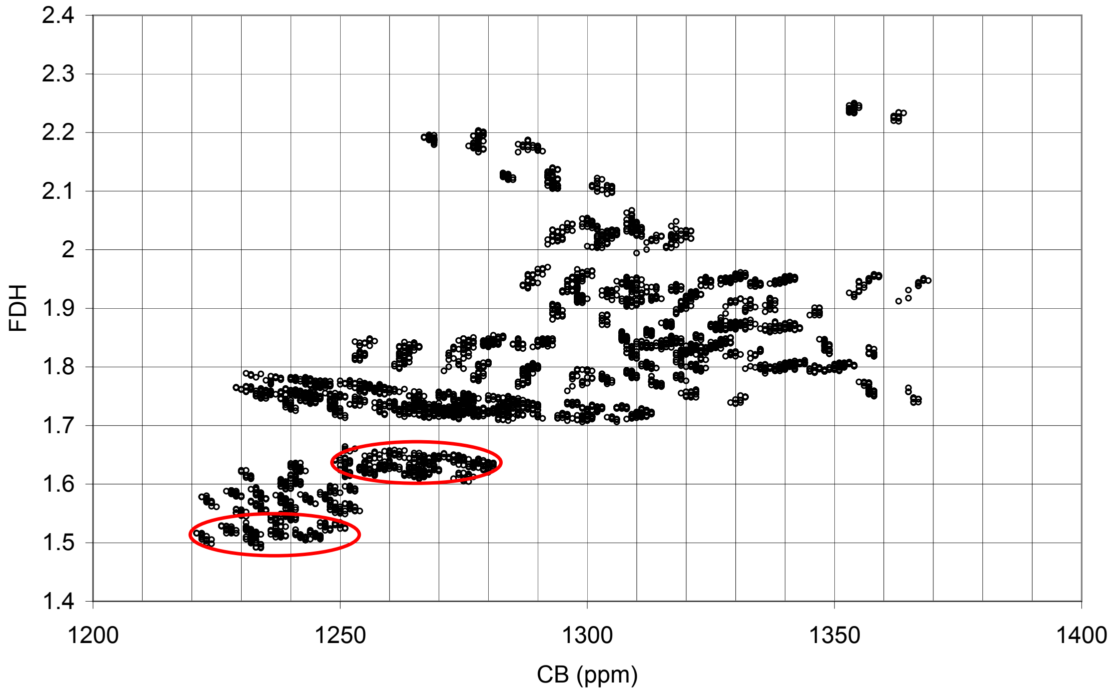

Figure 3. BOC Solutions for the 4175 4-Batch Patterns Enumerated Initially

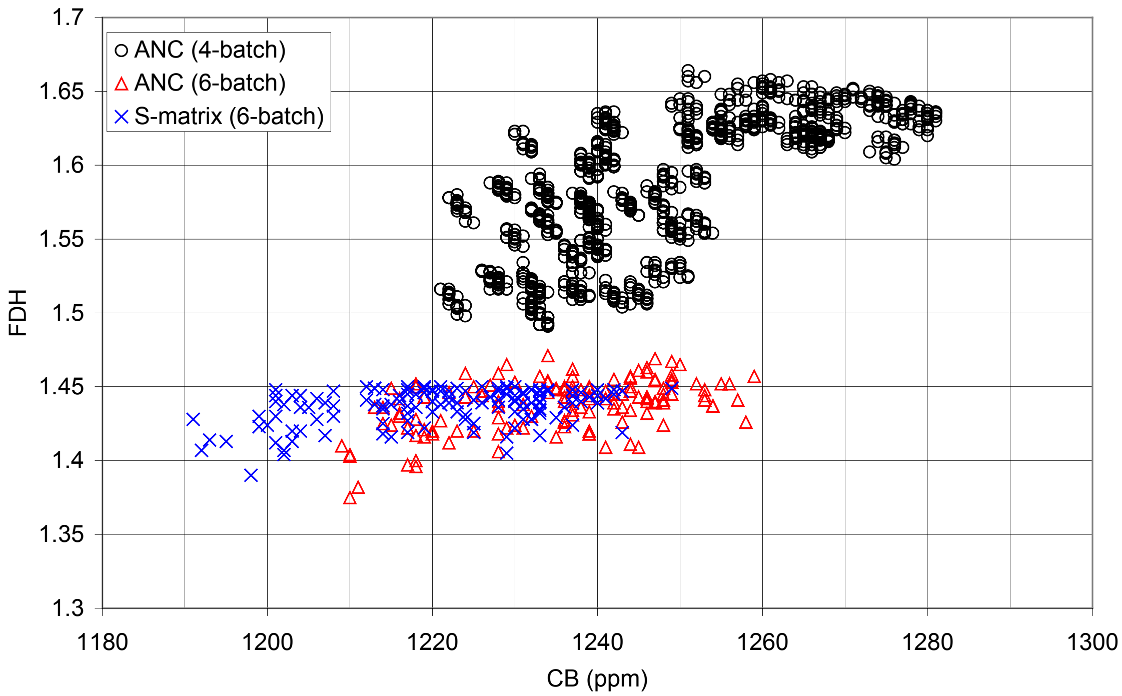

Figure 4. BOC Solutions for the 613 4-Batch BLPs and the 136 6-Batch BLPs

Then we go from 6-batch BLPs directly to real LP. Each one of the four burnt batches is split into unique individual assemblies. For each of the 136 6-batch BLPs, an S-matrix is generated, throughout depletion to the end of cycle, in order to calculate power distribution and core reactivity response to splitting a burnt batch into distinct individual assemblies. The S-matrix is input to the B&B mixed integer linear programming code to search for the best LP that has the maximum cycle length among all the real LPs that can sprout from a given 6-batch BLP. In this search the limit of 1.435 on the peak pin power is imposed. There are 109 real LPs resulting from this search. Figure 5 shows the solutions for the 109 LP at end of cycle (EOC), as predicted by both ANC and the S-matrix. The agreement between the ANC and the S-matrix predictions is excellent. The bias in boron concentration seen in Figure 4 now disappears because the burnup exposure spread in a batch becomes smaller as the batch size decreases.

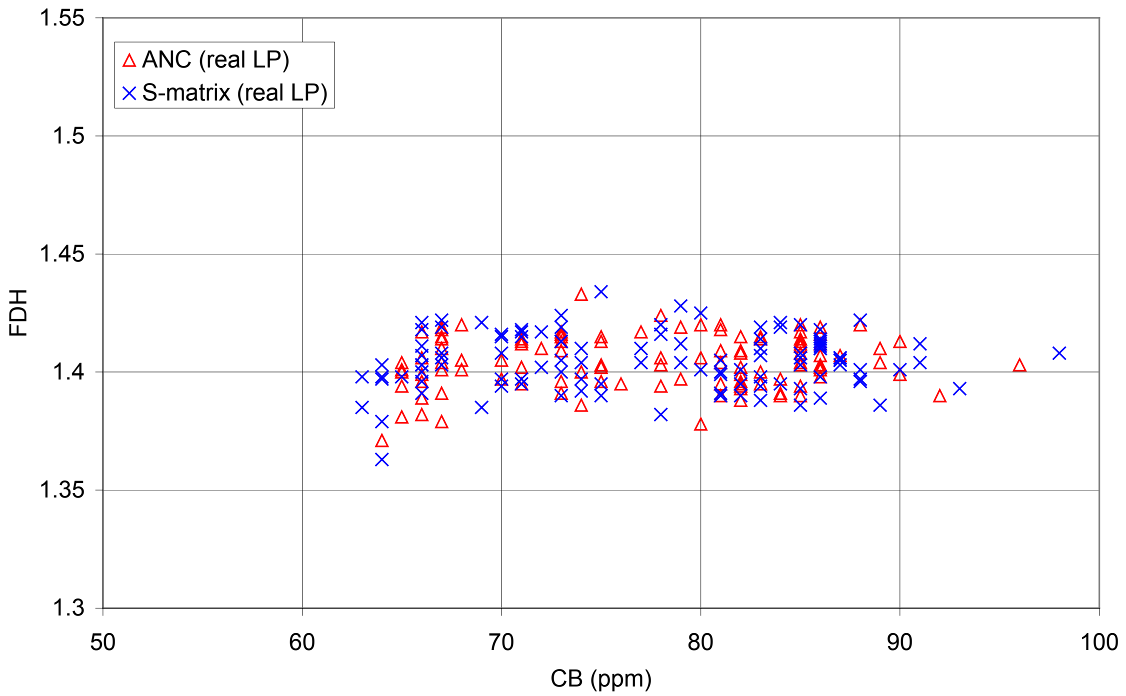

Figure 5. EOC Solutions for the 109 Real LPs

It should be emphasized that although all the 109 LPs satisfy the peak pin power limit, the cycle lengths vary significantly. From Figure 5 one can see an almost 40 ppm difference in the EOC critical boron concentration. This cycle length variation is to a large extent due to the different feed locations and the degree of low leakage of the initial 4-batch BLPs enumerated under the position constraints. Although in the initial enumeration we required the feeds to occupy about 60% of the peripheral locations, the exact feed locations on the periphery were not specified. The cycle length also depends on how many of the twice burnt assemblies versus the once burnt assemblies are placed in the other 40% of the peripheral locations. It is particularly appealing to see the large variety of acceptable LPs found, indicating clearly the effectiveness of the BPEC enumeration and the B3PEC search process employed by the LP-FUN code.

The above ANC results are all obtained using the 3D to 2D collapsed depletion model of ANC. It is important to confirm that the final LPs found by LP-FUN can be directly used for design. This is confirmed by comparing the 3D ANC results for the above 109 LPs to the final results of LP-FUN.

Figure 6 makes this comparison by displaying the prediction difference between 3D ANC and the 3D collapsed 2D ANC used by LP-FUN. Peak pin power and critical boron concentration for the 109 LPs are compared. As one can see the peak power difference is small throughout depletion. There is a small bias in critical boron concentration, which is depletion dependent. This bias is small, +10ppm at BOC and -5ppm at EOC. More importantly, the bias is "constant" in the sense of independent of the LPs. In fact the same bias is there in the original 3D to 2D collapsed model. Therefore this bias can be easily accounted for using the information from the original 3D to 2D collapsed model.

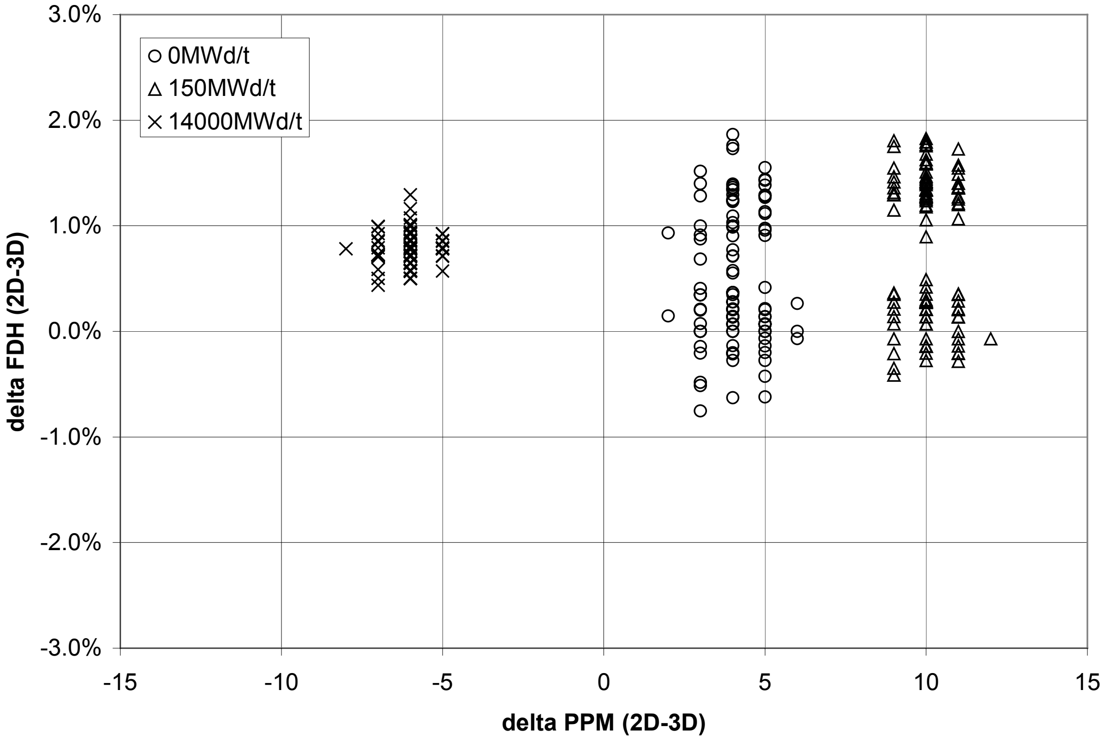

Figure 6. EOC Prediction Comparison Between 3D ANC and LP-FUN

3.3 PRACTICALITY OF LP-FUN

For an LP search code to be practically useful, a designer must be able to use it with confidence to solve real engineering design problems effectively in a predictable and acceptable running time. Solving it effectively means providing a sufficiently large number of good patterns that are sufficiently different and can all meet the designer's needs, so that the designer has realistic alternatives to evaluate. Using it with confidence means several things. First the found LPs must be able to survive analysis with the actual design code. Second the search process is robust and repeatable. Third the user has a good idea of to what extent the user has actually searched and to what degree the problem is solved. Predictable running time means that for the search that the user intends, the problem size and the running time can both be well estimated beforehand. Acceptable running time means that for a typical real design problem the running time is in hours or over night.

LP-FUN meets all these requirements. The search process described in Section 2 and the examples demonstrated in Section 3 show that the B3PEC method, albeit not examining the complete search space due to necessary constraints and approximations, can indeed provide the user with the confidence and effectiveness of the search. The examples given here just show some of the many flexible ways of using LP-FUN. Different types of problems can be handled with different strategies in enumeration, batch splitting and the associated depletion steps, burnable absorber optimization, and so on.

As for the running time, the B&B mixed integer linear programming is so fast that nearly all the time is consumed in the required 2D ANC calculation prior to each use of B&B mixed integer linear programming. For every BLP, at each batch-split level, the required number of 2D ANC calculation is known. Since one has a good idea of roughly how many BLPs will be tracked after the initial BOC screening and how many split levels and the associated depletion steps to track, the total number of 2D ANC calculation can be fairly well estimated and therefore the total running time. With today's computing capability in both design codes and computing machines, one can readily reach the conclusion that for typical design problems the running time is quite acceptable.

4. CONCLUSIONS

A new deterministic LP search method, B3PEC, has been developed. This method is built on two techniques, the technique of recursive pattern enumeration under position constraints and the B&B mixed integer linear programming. The method has the potential of searching comprehensively all the loading patterns in which a user is interested. Unlike stochastic search methods, where a user does not know how the search progresses, the B3PEC method enables a user to get a good idea of how the search is going on. The user defines the search domain and knows what part of the problem space has actually been searched and to what degree the problem is solved. The size of a search problem and its running time can both be pre-estimated by the user. The method can be implemented using a design code directly without the need of a separate module for neutron diffusion calculation.

The B3PEC method has been implemented in a new loading pattern search code, LP-FUN, using directly the Westinghouse core design code ANC. Applications to real design problems have confirmed the expected functions and performance. A variety of good patterns that all meet the user's requirements can be obtained effectively in predictable and acceptable running time, and the patterns can be used directly for core design.

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