OPTIMIZATION OF BWR FUEL LATTICE RADIAL DESIGN USING TABU SEARCH

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ABSTRACT

This paper describes an optimization method applicable to the design of radial enrichment and gadolinia distributions for boiling water reactor fuel assemblies. The method was coded in a computing system in which the optimization code uses the Tabu search method to select potential solutions and the HELIOS code to evaluate them. The goal of the method is to search an optimal fuel utilization looking for good reactivity performance and a low local peaking factor. Time-dependent-depletion effects were considered in the optimization. An application example is presented for a 10X10 fuel lattice with 10 different fuel types. Results of this time-dependent-depletion optimization are compared with an optimization at the beginning-of-life only.

I. INTRODUCTION

Radial fuel assembly design is one of the principal tasks in boiling water reactor (BWR) fuel management. A variety of constraints are imposed on the nuclear design of a BWR fuel assembly (FA) due to requirements of operation, safety and economy. The requirements include the reactivity for operation, the reactivity coefficients of void and Doppler, the control rod worth, etc. These reactivity characteristics are strongly dependent by the couple geometry and material conditions; e.g. fuel rod size and pitch, hydrogen-to-heavy-metal ratio, average fuel density and enrichment, control poison material and geometry, and so on. Other requirements are the shut-down margin (SDM) and the thermal margins (e.g. maximum linear heat generation rate (MLHGR)). These parameters are not determinable from the radial fuel assembly design alone; radial and axial fuel assembly design and the in-core load design significantly interact with each other. Thus, the detailed design of a BWR FA needs to be carried out simultaneously taking into account these interactions. The method described in this paper focuses on the problem of determining an optimal radial fuel enrichment and gadolinia distributions under given constraints. The axial FA design and the in-core load pattern design are outside the scope of this study. The study starts considering defined lattice geometry with fixed fuel rod size, pitch and water zones. The number of different pin enrichments to be utilized is restricted by manufacturing requirements.

Several works related to optimization of fuel rod enrichment distribution for BWR fuel assembly have contributed to the understanding and the solution of this large combinatorial problem. The method of

approximation programming [1] gives quite satisfactory results producing feasible candidate designs comparable to those elaborated by an expert engineer. A methodology that combines the response matrix method with non-linear programming techniques [2] was applied to search for an optimal pin enrichment distribution that gives the best approximation to a prescribed power distribution in two-dimensional FA. Other application to the optimization of MOX enrichment distributions in typical light water reactor assemblies [3] used a simplex method-based algorithm.

The Tabu search (TS) method, developed by Glover [4,5], is an iterative heuristic procedure for solving complex combinatorial optimization problems. TS is a constrained search procedure designed to overcome local optimality. In nuclear applications TS has been used for in-core reload pattern optimization in pressurized water reactors [6] and for the axial fuel assembly optimization in BWR [7] obtaining good results. Other work uses a simple linear perturbation method and a modified TS method to select potential optimized BWR load patterns [8]. Now, in this paper TS is applied to optimization of radial enrichment and gadolinia distributions for BWR fuel lattices.

2. METHODOLOGY

The TS optimization method was implemented to determine the "optimum" fuel enrichment and gadolinia distribution within a BWR fuel lattice. TS is based on the idea of moving step by step, from an initial candidate solution towards a solution giving the minimum value of some objective function, with a special feature designed to avoid being trapped by local minima. In order to evaluate the objective function, the optimization system was linked to the lattice code HELIOS [9] and executed in a Compaq Alpha Work Station. Then it is necessary to define a representation of the solution, the objective function and the implementation of the TS to the specific problem.

2.1. REPRESENTATION OF THE SOLUTION

The radial FA design is focused to find the optimal distribution of fuel types with different ^{235}U enrichment and gadolinia (Gd_2O_3) concentrations. Typical BWR fuel assemblies are 8X8, 9X9 or 10X10 pin arrays with a set of around of ten different fuel types. Consequently radial FA design is a complex combinatorial optimization problem, in which optimal fuel utilization is investigated searching for a good reactivity performance and a low local power peaking factor (PPF) during lattice exposition. A candidate solution $\boldsymbol{x}$ could be represented by a bi-dimensional array indicating the fuel composition located at each pin position in the lattice.

2.2. OBJECTIVE FUNCTION

The objective function was formulated to find the solution $\boldsymbol{x}$ with:

- the minimum average lattice enrichment;
- an average gadolinia concentration fixed as target Gd_{target}, at 0 MWd/T of exposure, to reduce the fuel reactivity excess;
- a PPF, at 0 MWd/T of exposure, smaller than a limit value PPF_{max};
- the best approximation to a prescribed infinite-multiplication-factor (k-infinite) as a function of exposure.

The first one is the objective of the optimization and the last three are the constraints. In order to accomplish the last constraint, the objective function minimizes the sum of the squared deviations, at different exposure steps **i,** between the k-infinite of the lattice $\boldsymbol{x}$ ($k_{inf\_i}(\boldsymbol{x})$) and the k-infinite target ($k_{inf\_target\_i}$). This is expressed in equation (1).

$$\sum_{i=0}^{N} \left(k_{inf\_i}(\boldsymbol{x}) - k_{inf\_target\_i} \right)^2 \qquad (1)$$

(2)

Equation (1) can be divided in two terms: the first for the step i = 0 MWd/T and the other for i = 1 to N exposure steps. This division permits the utilization of a strategy to reduce computing time in the optimization as explained in section 2.4.

Integrating all the terms, the objective function can be written as showed in the equation (2).

$$f(\mathbf{x}) = enrich(\mathbf{x}) \cdot w_1 + (k_{inf\_0}(\mathbf{x}) - k_{inf\_target\_0})^2 \cdot w_2 + (PPF(\mathbf{x}) - PPF_{max}) \cdot w_3 + \left| Gd(\mathbf{x}) - Gd_{target} \right| \cdot w_4 + \sum_{i=1}^{N} (k_{inf\_i}(\mathbf{x}) - k_{inf\_target\_i})^2 \cdot w_5 \quad (2)$$

Where:

$f(\mathbf{x})$	is the objective function to be minimized
$\mathbf{x}$	is the bi-dimensional array of fuel types related to the positions in the lattice
$enrich(\mathbf{x})$	is the average lattice enrichment for **x** solution
$k_{inf\_0}(\mathbf{x})$ and $k_{inf\_i}(\mathbf{x})$	are the k_{inf} at 0MWd/t and at exposure step i for **x** solution
$k_{inf\_target\_0}$ and $k_{inf\_target\_i}$	are the targets k_{inf} at 0MWd/t and at exposure step i
$PPF(\mathbf{x})$	is the local PPF for **x** solution
PPF_{max}	is the local PPF limit
$Gd(\mathbf{x})$	is the average lattice gadolinia concentration for **x** solution
Gd_{target}	is the target average lattice gadolinia concentration
i	is the exposure step
N	is the total number of exposure steps
w_1 to w_5	are user-defined weighting factors to accommodate design with preferences.

The Enrichment($\mathbf{x}$) and the Gd($\mathbf{x}$) are calculated considering the composition of the fuel types and their presence in the lattice $\mathbf{x}$, the $k_{inf\_0}(\mathbf{x})$, $k_{inf\_i}(\mathbf{x})$ and PPF($\mathbf{x}$) are obtained executing the HELIOS code.

2.3. TABU SEARCH METHOD DESCRIPTION

Briefly, the TS method minimizes $f(\mathbf{x})$, subject *to* $\mathbf{x}$ in X, where f is a cost function, and X is a set of candidate solutions. It starts from an *initial candidate solution* and tries to reach a global minimum by moving from one candidate solution to another. To accomplish this, a set M of simple modifications must be defined. These modifications are called *moves*, which can be applied to a given candidate solution to move to another. The notation $\mathbf{x}' = m(\mathbf{x})$, m in M, indicates that m transforms $\mathbf{x}$ into $\mathbf{x}'$. This leads to the definition of a *neighborhood*. For each candidate solution $\mathbf{x}$, the neighborhood $N(\mathbf{x})$ is the set of all candidate solutions directly reachable from x by a single move m in M. When $N(\mathbf{x})$ is large, at each step of the iteration process, a subset V^* of $N(\mathbf{x})$ is generated and the move is made from x to the best solution x^* in V^*, whether or not $f(\mathbf{x}^*)$ is better than $f(\mathbf{x})$.

Up to this point, the algorithm is close to a local improvement technique, except that the move from $\mathbf{x}$ may be to a worse solution $\mathbf{x}^*$, and thus may escape from local minimum of f. To prevent cycling, any move that returns to any local optimum recently visited is *tabu* (forbidden). This is accomplished in a *short-term memory function* by storing the tabu move in a set T, called the *tabu recency list*, of length t. A move will remain tabu during t iterations, so the tabu list can be represented by a queue: at each iteration the opposite move from $\mathbf{x}$ to $\mathbf{x}^*$ is added at the end of T, while the oldest is removed from T. Another mechanism that can improve performance is diversification, that is, to encourage the method to search unexplored regions. This mechanism can be achieved by implementing a *long-term memory*

function via frequency counts, a *tabu frequency list*. A move is forbidden if its frequency exceeds a *maximum frequency value*, and this tabu frequency list and its limit value must be updated dynamically when iterations increase. These tabu lists could forbid certain interesting moves, such as moves that lead to a better solution than the *best* one found so far. Considering this, the *aspiration criterion* is introduced to cancel the tabu status of a move when it is judged to be a useful move.

Stopping rules must also be defined, it can be a fixed lower value f*, or it can be a fixed number of possible solutions investigated, or it can be a fixed number of iterations.

2.4 APPLICATION TO THE RADIAL FUEL DESIGN

The method described above was applied to a 10X10 fuel pin array with two water zones and diagonal symmetry. The fuel used is dioxide of uranium (UO2) and some fuel are mixed with gadolinia (Gd_2O_3) as burnable poison.

The lattice is simulated using the HELIOS code with the fuel pins, the water regions, the channel and the control rod explicitly represented in two-dimensions. In HELIOS this lattice can be represented using half diagonal symmetry with 51 pin positions as showed in Figure 1. Having account manufacturing requirements, only ten different pin enrichments and gadolinia concentrations were utilized in this application (see Table I).

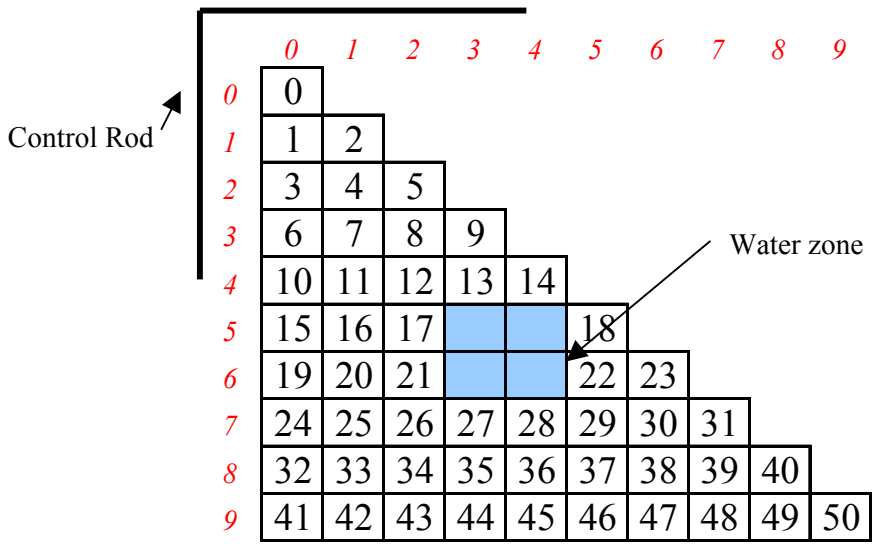

Figure 1. Schematic representation of the lattice.

Table I. Fuel enrichments and gadolinia concentrations in fuel types considered.

	UO_2-1	UO_2-2	UO_2-3	UO_2-4	UO_2-5	UO_2-6	Gd-1	Gd-2	Gd-3	Gd-4
U_{235} %w	2.0	2.8	3.6	4.4	3.95	4.9	3-95	4.4	4.4	4.4
Gadolinia %w	0	0	0	0	0	0	5	5	4	2

In order to accelerate and simplify the search, some heuristic rules can be applied in the implementation. In this study only three heuristic rules were applied:

- first, the lowest enriched fuel (UO_2-1) is only used at the corner positions: (0,0), (0,9) and (9,9)

- second, fuels containing gadolinia cannot be placed in the lattice's edge;
- third, water region position is fixed.

This means that there are 16 pins that could have 5 different fuels, and there are 32 pins that could have 9 different fuels. The resulting number of total combinations is $5^{16} + 9^{32} = 3.44E+30$ and then the search must be well conducted to reduce computing time.

In this study, the first candidate solution $\boldsymbol{x}$ was constructed using the annalist's experience. This solution is considered the first current solution in the search procedure. The TS method is designed to select, at each iteration, the best available *move* using the current solution $\boldsymbol{x}$. Two types of *moves* were defined: a move *type 1,* which allows a change in the quantity of different fuels in the lattice; and a move *type 2,* that allows to modify the pins' distribution in the FA and to find the best location of a certain fuel type in the lattice. A move *type1* is entirely defined by the random selection of one pin position and the random selection of one fuel composition from the list of available fuel types. A move *type 2* is a pair wise exchange (or swap), it exchanges the location of two different fuel compositions in the lattice and is defined by the random selection of two positions having different fuel types. In this application a maximum number of moves in an iteration step was fixed, but the process stops the moves when a solution $\boldsymbol{x}^*$ is better than solution $\boldsymbol{x}$. To prevent cycling, the opposite move from $\boldsymbol{x}$ to $\boldsymbol{x}^*$ is added at the end of *T*, while the oldest is removed from *T.* The aspiration criteria cancel the tabu status of a move when it finds a solution with a better function value than the best solution in the past. The process stops when a number of iterations is reached or the user can stop it manually.

Taking in to account that a time-dependent-depletion (TDD) evaluation using HELIOS consumes a considerable computing time; in order to reduce computing time, the TS implementation uses the next strategy: each candidate solution is firstly evaluated at 0 MWd/T and while the solution is far from having a reactivity near the $k_{inf\_target\_0}$ or the $PPF(\boldsymbol{x})$ exceed a lot the PPF_{max}, then this lattice is considered *unacceptable* and the TDD calculations are not done and not considered in the objective function of the TS process. Once an *acceptable* solution is found, the TDD calculations are done and their results are considered in the objective function. After that, each candidate solution that is not closer enough to the target values is eliminated before the TDD evaluation; then another solution is generated to replace it. This action is implemented using a PPF_{start} and a $k\text{-}deviation_{start}$. A candidate solutions with a

$$PPF(\boldsymbol{x}) > PPF_{start}$$

or

$$k\text{-}deviation = abs\left(k_{inf\_0}(\boldsymbol{x}) - k_{inf\_target\_0}\right) / k_{inf\_target\_0} > k\text{-}deviation_{start}$$

is considered an unacceptable solution. Figure 2 shows the TS technique adapted to the BWR FA radial optimization problem in which this strategy has been incorporated. A *partial* evaluation is only a 0 MWd/T and a *full* evaluation includes the TDD calculations.

A reference lattice was selected in order to determine the target design parameters. Table II shows the pre-established reactivity, as function of the exposure, imposed as constraint. It can be noticed that the last burnup step is at 20000 MWd/T when the gadolinia is totally depleted. For the lattice studied in this work, a HELIOS calculation at only 0 MWd/T uses approximately 0.21 CPU minutes. In order to reduce computing time TDD calculations stop at 20000 MWd/T of exposure, using approximately 1.6 CPU minutes.

Table II. Reactivity targets as a function of exposure.

Exposure Step	1	2	3	4	5	6
MWd/T	0*	0	1000	12000	15000	20000
K-infinity	1.02929	1.0087	1.01129	1.13167	1.4318	1.11704

*No Xenon

Table III shows the maximum PPF limit imposed as constraint at 0 MWd/T, and the gadolinia concentration target imposed as constraint at 0 MWd/T. The limits PPF_{start} and k-$deviation_{start}$ used to activate the TTD evaluations are also showed in table III. Table IV shows general data associated to the TS characteristics.

Table III. Pre-established design constraints and start limits

PPF_{max} at 0 MWd/T	1.438
Gd_{target} at 0 MWd/T	0.81522 %
PPF_{start}	1.47
k-$deviation_{start}$	0.6 %

Table IV. Tabu Search characteristics.

Maximum number of moves in each iteration	30
Maximum number of iterations	200
Size of the tabu list	15

3. RESULTS AND DISCUSSION

To highlight the importance of the incorporation of the TDD effects in the objective function, a TDD optimization case was compared with a beginning-of-life (BOL) optimization case. A total of 72 iterations were done, that includes a total of 1351 evaluations, and only 755 of them included a TDD evaluation. The best solution was found at the iteration 51, which is the evaluation number 727. The user stopped the process manually at cycle 72 cars the best solution could not be improved since the cycle 51. All HELIOS calculations were done at 40 % of voids and a fuel temperature of 520 ºC. The results obtained for the final best solution of the TDD optimization are presented in Table IV. This table also includes the results for the best solution of a BOL optimization process not including TDD effects, and the data of the reference lattice. The BOL optimization gave a very low average enrichment. Nevertheless, when TDD effects were calculated, it was found a high sum of quadratic k-deviation value. Then it is observed that the inclusion of TDD effects in the optimization is recommended to assure a good reactivity performance during lattice exposure.

Table IV. Results for the best solution at the end of the optimization.

Lattice	k-infinite at 0 MWd/T	PPF at 0 MWd/T	Gadolinia w % at 0 MWd/T	Sum of quadratic k-deviation	Enrichment w %
TDD Optimization	1.02935	1.438	0.81522	3E-06	4.0560
BOL Optimization	1.02912	1.419	0.81522	3.1E-04*	3.9435
Reference	1.02929	1.439	0.81522	0	4.1065

* This parameter was calculated when the optimization process had finished.

Figure 3 shows the evolution of the objective function, for the best solution and for all solutions fully evaluated, as a function of the solution number. A good convergence is appreciated. Figure 4 shows the evolution of the k-infinite at 0 MWd/T versus the solution number; it includes values for the current solution, the best solution, the target, all solutions evaluated at 0 MWd/T (All 0) and all

solutions with TDD evaluation (All). The k-infinite upper and lower limits used to start a full evaluation are also indicated. Figure 5 presents the evolution of the PPF evaluated at 0 MWd/T as a function of the solution number, it includes values for the current solution, the best solution, the PPF_{max}, all solutions partially evaluated (All 0) and all solutions fully evaluated (All); the PPF_{start} used to activate a full evaluation is also indicated. Figure 6 shows the evolution of the sum of quadratic k-deviation as a function of the solution number, the best and all fully evaluated solutions.

The evolution of the average lattice enrichment for the current and the best solutions is showed in Figure 7. In Figure 8 the k-infinite as a function of the lattice exposure is plotted for the best solution obtained at the end of the TDD optimization process and for the reference lattice; also the results for the best solution obtained at the end of the BOL optimization process were included in this figure. It is observed that the BOL optimization does not satisfies the requirements of reactivity as function of burnup. Table V summarizes the characteristics of the best solutions for the TDD and BOL optimizations, and the reference lattice. The best solution obtained at the end of the TDD optimization is presented in figure 9.

Table V. Characteristics of the best and reference lattices.

Lattice	UO_2-1	UO_2-2	UO_2-3	UO_2-4	UO_2-5	UO_2-6	Gd-1	Gd-2	Gd-3	Gd-4
TDD	4	6	11	16	22	17	11	2	2	1
BOL	4	12	14	10	22	14	10	1	5	0
Reference	4	6	6	32	16	12	4	9	2	1

CONCLUSIONS

Two conclusions can be highlighted: first, this work has shown that the tabu search technique is applicable to optimize the fuel enrichment distribution in a BWR fuel assembly subject to reactivity and power peaking factor as constraints; and second, time-dependent depletion calculations are indispensable to be included in the objective function. It is also clear that the objective function could incorporate additional parameters to do more robust the lattice evaluation performance. For example control rod introduction, void effects, temperature effects, etc. could be incorporated On the other hand, several strategies to reduce computing time can be implemented without penalize the results accuracy.

ACKNOWLEDGMENTS

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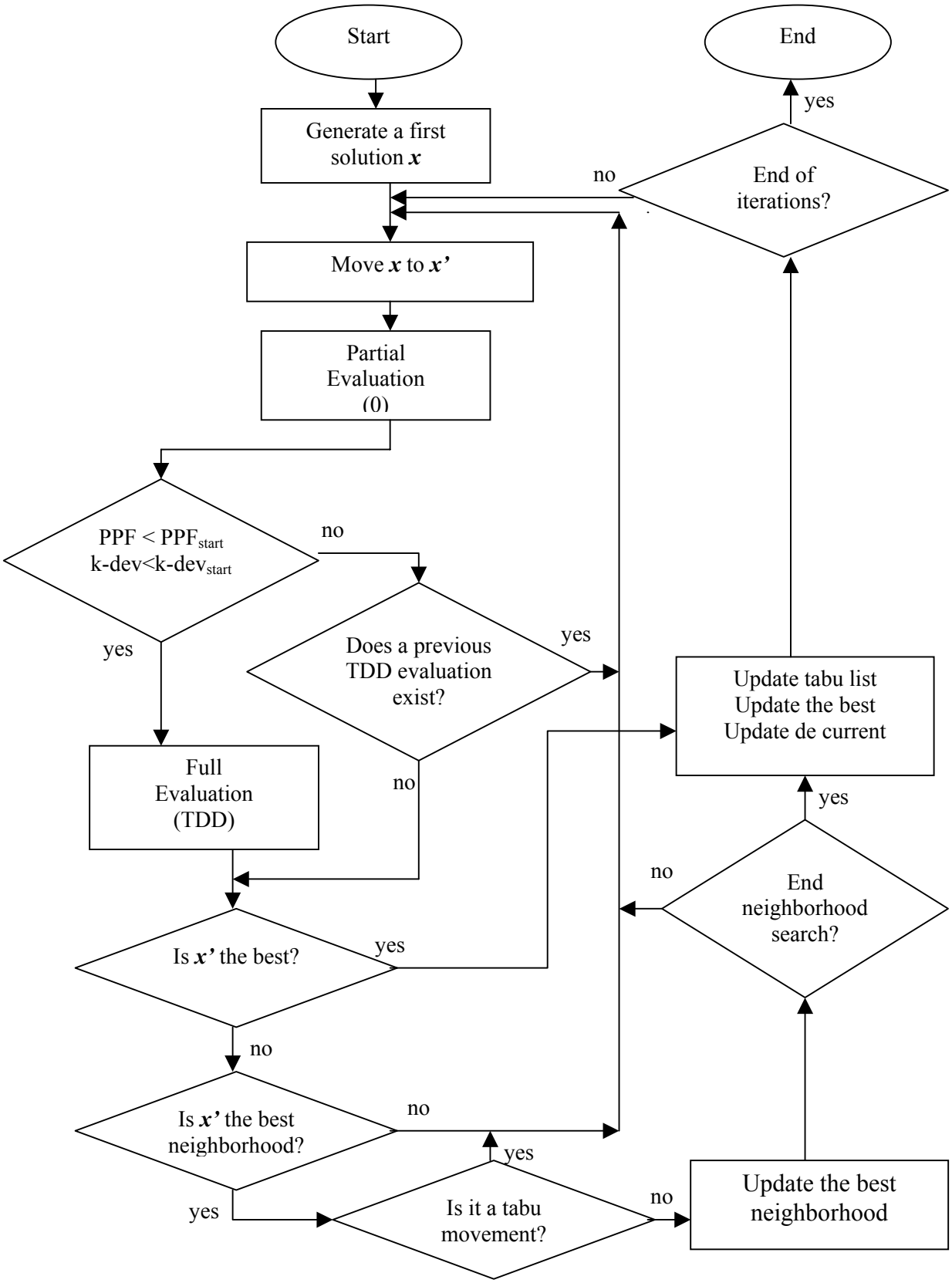


Figure 2. Diagram of the Tabu search implementation.

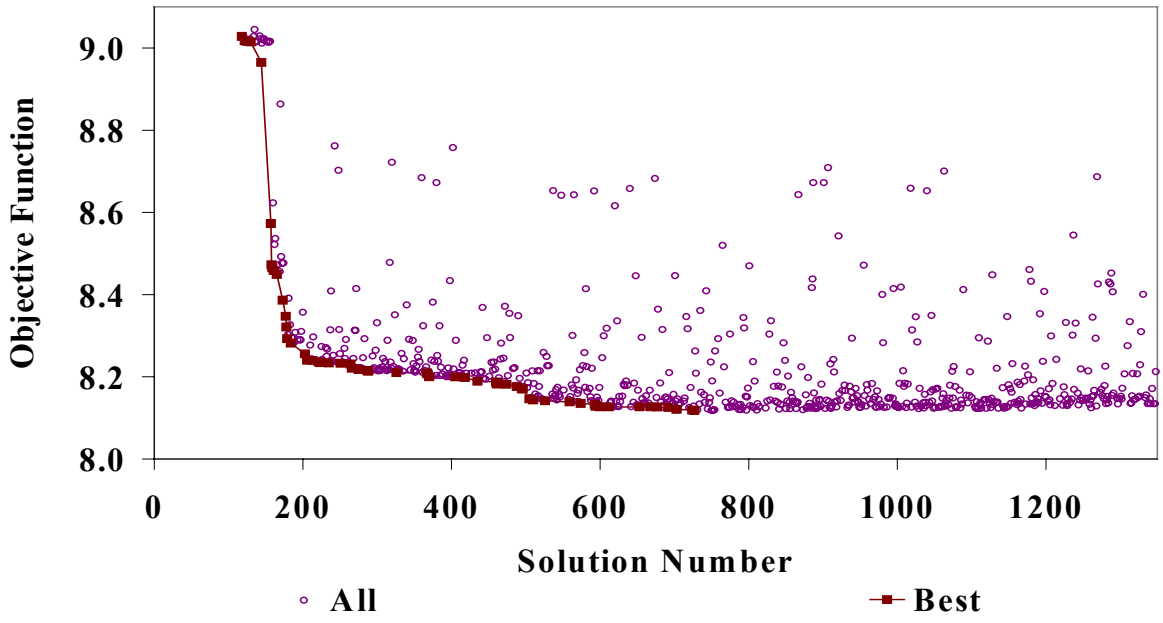


Figure 3. Objective function evolution in the TDD optimization.

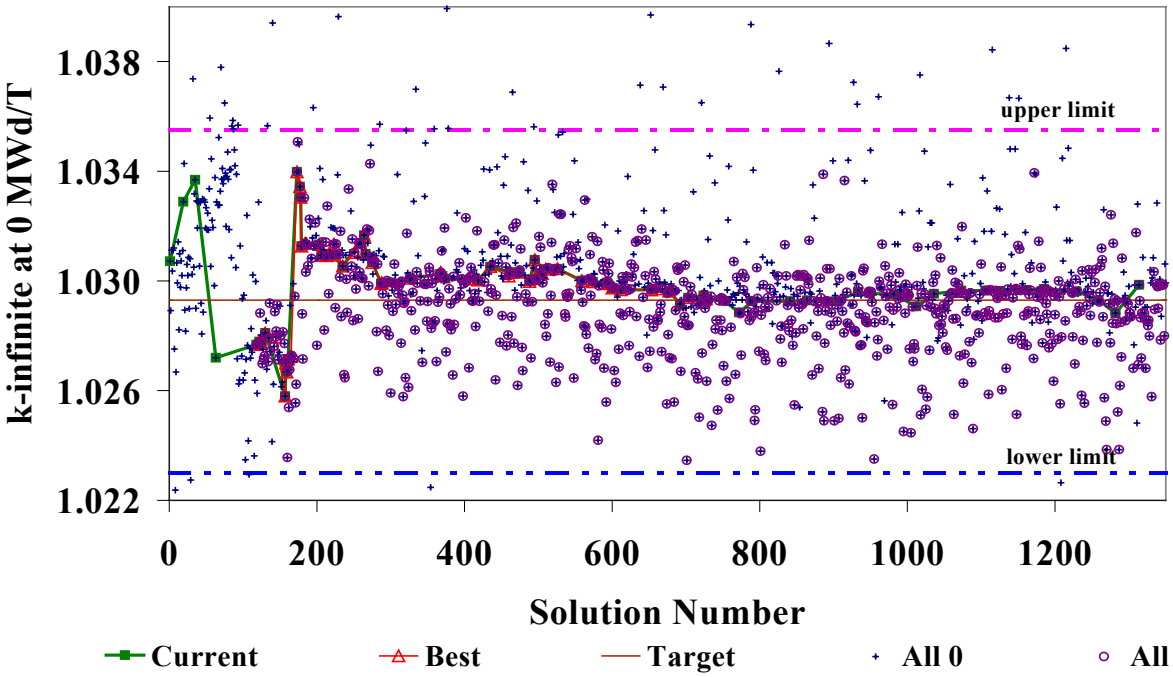


Figure 4. k-infinite (at 0 MWd/T) evolution in the TDD optimization.

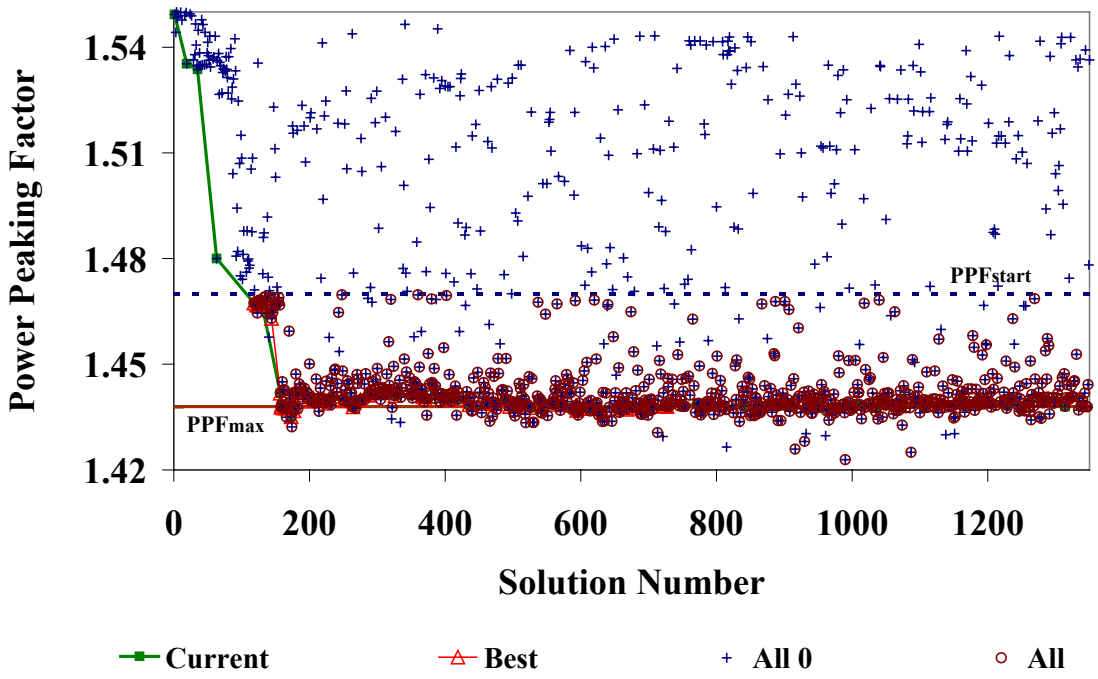


Figure 5. PPF (at 0 MWd/T) evolution in the TDD optimization.

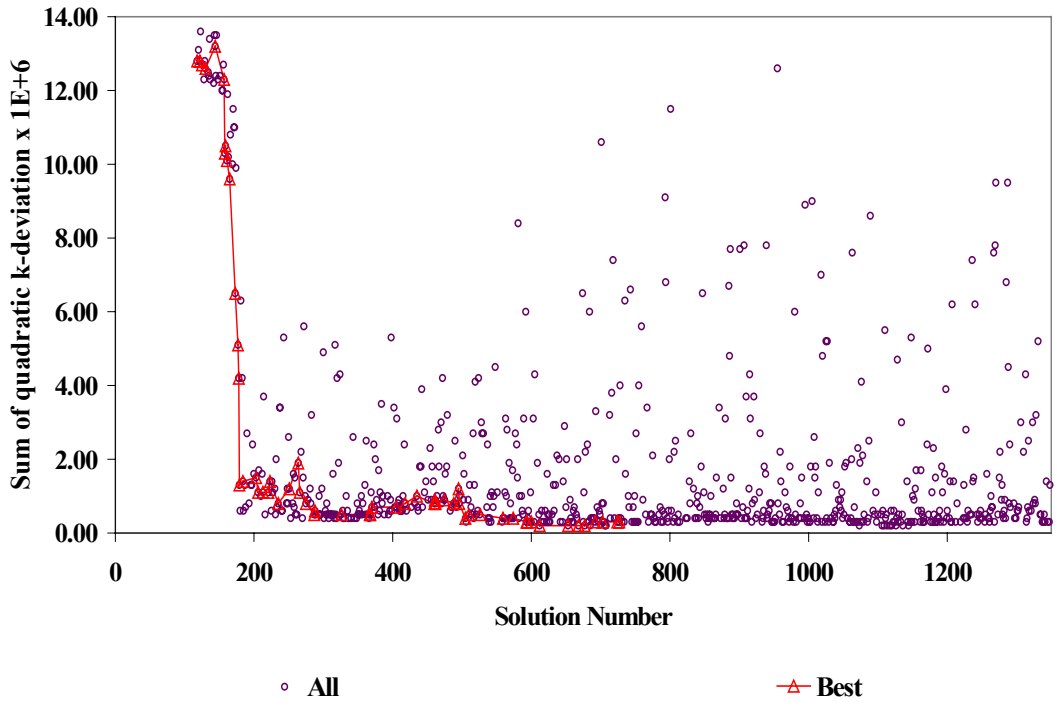


Figure 6. Sum of quadratic k-deviation evolution in the TDD optimization.

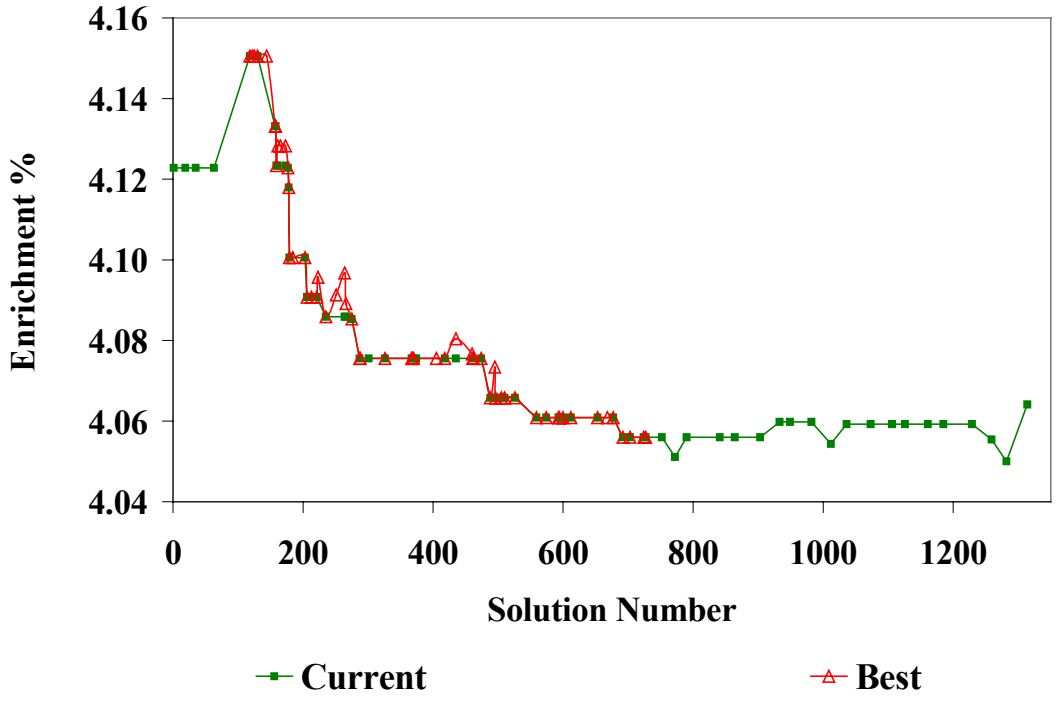


Figure 7. Evolution of the lattice average enrichment in the TDD optimization.

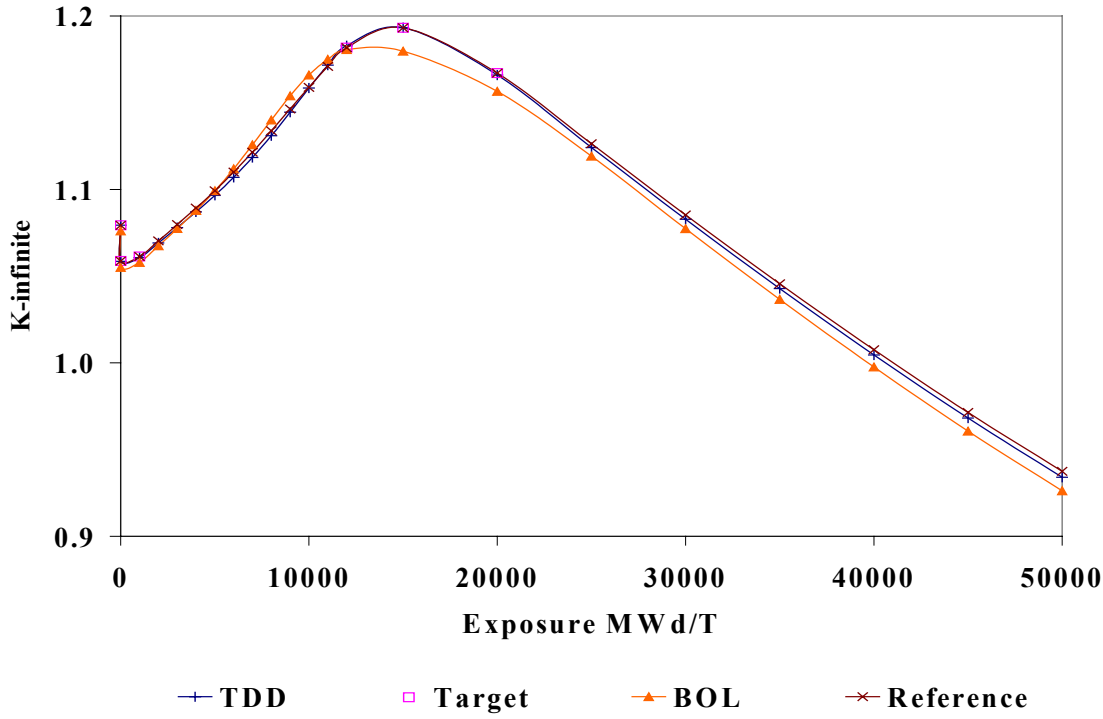
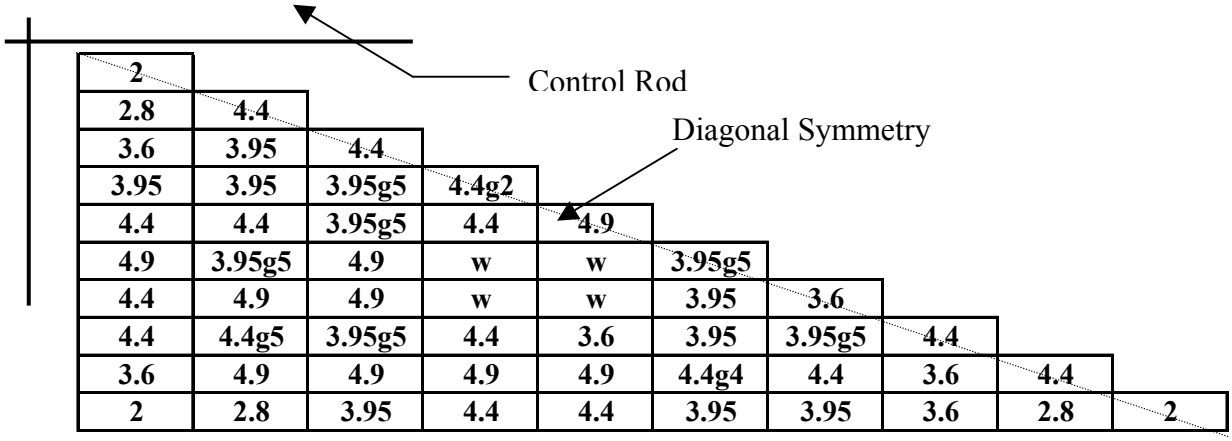


Figure 8. k-infinite as function of exposure for the best solutions.



2									
2.8	**4.4**								
3.6	**3.95**	**4.4**							
3.95	**3.95**	**3.95g5**	**4.4g2**						
4.4	**4.4**	**3.95g5**	**4.4**	**4.9**					
4.9	**3.95g5**	**4.9**	**w**	**w**	**3.95g5**				
4.4	**4.9**	**4.9**	**w**	**w**	**3.95**	**3.6**			
4.4	**4.4g5**	**3.95g5**	**4.4**	**3.6**	**3.95**	**3.95g5**	**4.4**		
3.6	**4.9**	**4.9**	**4.9**	**4.9**	**4.4g4**	**4.4**	**3.6**	**4.4**	
2	**2.8**	**3.95**	**4.4**	**4.4**	**3.95**	**3.95**	**3.6**	**2.8**	**2**

Figure 9. Lattice description for the best solution in the TDD optimization.