BURNUP IMPORTANCE FUNCTION AND ITS APPLICATION TO OECD/NEA/BUC PHASE II-A AND II-C MODELS

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ABSTRACT

As the burnup proceeds, reactivity of fuel assemblies for light water reactors decreases by depletion of fissile nuclides, especially in the axially central region. In order to describe the importance of the end regions to the reactivity increase as fuel burnup proceed, a burnup importance function was introduced as a weighting function to a local burnup variation contributed to a reactivity decrease. The function was applied to the OECD/NEA/BUC Phase II-A model, which represent irradiated pressurized water reactor (PWR) fuel rods, and a simplified Phase II-C model, which studies reactivity changes due to local variations of burnup. The application to Phase II-A model clearly showed that burnup importance of the end regions increases as burnup and/or cooling time increases. Comparison of the burnup importance function for different initial enrichments was examined. The application to the simplified Phase II-C model was also made. The result showed that the burnup importance function was helpful to find the most reactive fuel burnup distribution under the conditions that the average fuel burnup was kept constant and the variations in the fuel burnup were within the maximum and minimum measured values.

1. INTRODUCTION

For the system involved in light-water reactor fuels, the total reactivity decreases as fuel burnup proceeds except when neutron poison is depleted for an initial period by absorption of neutrons. “Burnup credit” is the terminology used for the criticality safety management of spent fuel that assumes this reactivity decrease.

Uniform burnup of fuel is often assumed for criticality safety evaluation for simplicity. The end effect is defined as a difference between the values of the neutron multiplication factor that assumes axial burnup profile and that disregards it.[1]

$$\Delta k_{eff} = k_{eff}(\text{profile}) - k_{eff}(\text{uniform}) \tag{1}$$

It is negative or very small positive for low burnup (typically less than 20 GWd/tU), and considerably large for higher burnup (typically more than 30 GWd/tU). This tendency of the end effect as a function of fuel burnup is interpreted to mean that the end regions of fuel rods, therefore also, of a fuel assembly, become relatively more important than the central part as the burnup of fuel proceeds. This paper intends to establish a local quantity that characterizes a degree of importance of fuel burnup to reactivity in order to study the end effect more profoundly.

2. BURNUP IMPORTANCE FUNCTION

2.1 DEFINITION

Although reactivity ρ (= 1- 1/k_{eff}) of a spent nuclear fuel system typically decreases as fuel burnup proceeds, burnup effect on reactivity of the fuel system is different from place to place of the fuel. If a region of fuel is very reactive and it dominates the reactivity of the system, an increase in burnup of that region should affect much on the reactivity. Burnup importance (BI), $I_{\mathrm{B}}(\mathbf{r})$, at a position **r** represents such a quantity, which is introduced by the following relation:

$$-\delta\rho = \int d\mathbf{r} I_{\mathrm{B}}(\mathbf{r}) \cdot \delta B(\mathbf{r}), \tag{2}$$

where $\delta B(\mathbf{r})$ is a slight change in fuel burnup at a position **r**, and $-\delta\rho$ is a reactivity decrease induced by an increase of fuel burnup $\delta B(\mathbf{r})$. For an application of the FI to a simplified Phase II-C model, the integration in Eq. (2) will be constrained that the total burnup should be unchanged.

2.2 EXPLICIT EXPRESSION

Reactivity of the system is expressed as the following, where $B(\mathbf{r})$ is burnup of fuel at a position **r**. Other notations follow the conventional reactor physics:

$$\rho = 1 - \frac{1}{k_{eff}}, \tag{3a}$$

$$= 1 - \frac{\int d\mathbf{r} \sum_{g,g'} \Phi_g^+ \left[\left(\Omega \cdot \nabla + \Sigma_{\mathrm{t},g} \right) \delta_{g,g'} - \Sigma_{\mathrm{s}g'g} \right] \Phi_{g'}}{\sum_{g,g'} \int d\mathbf{r} \Phi_g^+ \chi_g \nu_{g'} \Sigma_{\mathrm{f}g'} \Phi_{g'}}, \tag{3b}$$

where g and g' are numbers of energy groups.

Considering the 1st order variation $\delta B(\mathbf{r})$ in local burnup of Eq. (3b), and comparing the result with Eq.(2), the burnup importance function I_B(r) can be found to be expressed as:

$$I_{\mathrm{B}}(\mathbf{r}) = -\sum_{g} \Phi_g^{+} \left[\frac{1}{k_{eff}} \sum_{g'} \frac{\partial(\chi_g \nu_{g'} \Sigma_{\mathrm{f}g'})}{\partial B} \Phi_{g'} - \frac{\partial \Sigma_{\mathrm{R}g}}{\partial B} \Phi_g + \sum_{g' \neq g} \frac{\partial \Sigma_{\mathrm{s}g'g}}{\partial B} \Phi_{g'} \right] \Bigg/ \sum_{g,g'} \int d\mathbf{r} \Phi_g^{+} \chi_g \nu_{g'} \Sigma_{\mathrm{f}g'} \Phi_{g'} \qquad (4)$$

The burnup importance function $I_B(\mathbf{r})$ was utilized to express a decrease rate of the neutron multiplication factor k_{eff} by the averaged burnup $\overline{B}$ of the fuel region:

$$-\frac{dk_{eff}}{d\overline{B}} = \int d\mathbf{r} k_{eff}^2 I_{\mathrm{B}}(\mathbf{r}) \frac{dB(\mathbf{r})}{d\overline{B}} \qquad (5)$$

The integrand of the right-hand-side of Eq. (5) will be called "burnup importance" hereafter, and it will be numerically calculated.

2.3 COMPUTATIONAL FLOW

A computer program to calculate the right-hand-side of Eq. (5) was developed, which implements the MKENO-BURN code [2] to get nuclide concentrations and the DANTSYS code system [3] to obtain reactor physics quantities. A flow of this computer program is schematically shown in Figure 2. The forward and adjoint neutron fluxes as well as the macroscopic cross sections are required to calculate the burnup importance.

3. APPLICATION TO OECD/NEA/BUC PHASE II-A MODEL

3.1 OUTLINE OF PHASE II-A BENCHMARK

The Phase II-A Benchmark of OECD/NEA was intended to study the effect of an axial burnup profile on criticality calculation for a simple pin-cell model of a pressurized water reactor (PWR) fuel. Nuclide compositions were assumed in the benchmark.

It was an infinite array of typical PWR spent fuel rods immersed in water. Radial and axial dimensions of the fuel rod system are described in Figures 3 (a) and (b), respectively. The burnup profile was assumed to be symmetric to the midsection. The parameters changed for the benchmark were cooling time, average burnup, considering fission products or not, and considering burnup profile or not, which were shown in TABLE I as well the case numbers (in **bold**) studied in this paper.

3.2 END EFFECTS

The results of Phase II-A Benchmark showed that the end effect becomes prominent as burnup and/or cooling time increases. The fission products provide a positive component to the end effect since the accumulation of fission products is smaller at both ends than the central region and this makes the end regions more reactive.

The importance of end regions in the aspect of criticality safety increases as burnup proceeds. Figure 4 compares the burnup importance as a function of a height from the rod center for three cases, **1**, **6** and **7** specified in TABLE I, i.e., for the mean burnup of 0, 30 and 50 GWd/tU, respectively, the cooling time of one year, and considering fission products. The figure shows the following:

1. For the fresh fuel case, the central region dominates the burnup importance.
2. As the burnup proceeds, the burnup importance of the central region decreases. This is intimately related to depletion of ^{235}U in the central region. Accordingly, the end regions relatively increase their burnup importance and dominate.

Figure 5 compares the burnup importance as a function of the height from the rod center for the cooling time of 0, 1, 5 and 20 years in a case of 30 GWd/tU burnup. It indicates that the burnup importance of end regions increases as cooling time. This is because that longer the cooling time becomes, the β^--decay of ^{241}Pu to ^{241}Am (a half-life: 14.4 y) becomes more dominant in the central region than the end regions, where the burnup importance relatively increases as cooling time.

The above results were limited to the cases with the initial enrichment of ^{235}U was 4.5 wt.%. In order to see the dependencies on the initial fuel enrichment, we have additionally made calculation for the initial enrichment of 3.6 wt.%. The results for the initial enrichment of 3.6 wt.% are shown in Figure 6 in dark-gray curves whereas those for 4.5 wt.% cases are represented in light-gray curves. The difference in the initial enrichment did not produce much difference for the burnup of 0 and 50 GWd/tU cases. However, for the burnup of 30 GWd/tU case, the end effect in the 3.6 wt.% case was more profound compared to the 4.5 wt.% case. This tendency is interpreted that the depletion of ^{235}U becomes prominent in earlier burnup stages for a lower initial enrichment case than a higher initial enrichment case.

4. APPLICATION TO OECD/NEA/BUC PHASE II-C MODEL

4.1 OUTLINE OF PHASE II-C BENCHMARK

The Expert Group on Burnup Credit of OECD/NEA conducted Phase II-C Benchmark criticality calculation [5] based on the realistic burnup distributions of PWR assemblies. The purpose of the benchmark study was to study the sensitivity of criticality calculations of PWR assemblies to the axial burnup shape.

The Phase II-C Benchmark was based on the analysis of samples containing 850 axial burnup profiles derived from in-core flux measurement during the operation of the Nuclear Power Plant Neckarwestheim in Germany. Each axial burnup profile was approximated by a 32-equidistant-node representation and normalized to the average burnup profile to obtain burnup shapes. At these 32 nodes, averages and standard deviations of the shapes (local burnup divided by the average burnup) were calculated over the samples. The average and standard deviation of normalized axial burnup shapes of the samples are shown in Figure 7.

The minimum and maximum values at the six top nodes were determined depending on the average burnup (32 or 50 GWd/tU). The geometry of the proposed Phase II-C Benchmark was close to the one studied in Phase II-B (transport cask of 21 assemblies submerged in water, [6]). The effect of axial burnup asymmetry was studied through the variation of the shapes (essentially, a variation of the burnup ratios for the top six nodes from their minimum to their maximum values).

4.2 EFFECT OF VARIATION IN BURNUP DISTRIBUTION

The neutron multiplication factor, k_{eff}, was calculated for both 32 and 50 GWd/tU burnup cases with a combination of the continuous energy particle transport code MCNP4B2 based on the Monte Carlo method and the Japanese Evaluated Nuclear Data Library JENDL-3.2 [4]. Table II lists the calculation results for 32 GWd/tU burnup cases in terms of the relative k_{eff} to that assumed the average axial burnup shape, where the calculated k_{eff} = 0.876. In the table, case number IDs are specified in a form $n_1n_2n_3n_4n_5n_6$, where n_i (i = 1, 2, ..., 6) goes from 1 to 3. The meaning of n_i is the following: n_i = 1, 2, and 3 means that the n_i-th top node has the minimum, mean, and maximum burnup, respectively, of the measured 850 samples at that node. The mean burnup values from 18th to 29th nodes were adjusted to conserve the average burnup of 32 GWd/tU. The table shows that the k_{eff} will be most increased (1.5%Δk/k) if the top five or six nodes have their minimum burnup values, and it will become most decreased (-2.0%Δk/k) if the top six nodes have their maximum burnup values.

From the limitation of the program, the present calculation scheme was applied to a simplified Phase II-C model, which is an infinite array of fuel pins. The k_∞ of the model for the average burnup shape (denoted as 222222 case according to the previously defined notation) was 0.727, quite different from the k_{eff} value, 0.876, for the corresponding case of Phase II-C Benchmark. However, relative changes in the neutron multiplication factor are found in a fairly good coincidence for various burnup variations.

Burnup importance for the average burnup shape was calculated, and shown as a peaked curve in Figure 8. The figure indicates that the burnup importance has large values from the 2nd to 4th nodes. When one increases or decreases the burnup of these nodes, the neutron multiplication factor decreases or increases, respectively. The expected tendencies were really observed in Table II, which implies that the burnup importance is helpful to find the most reactive burnup profile under the condition that the average burnup kept conserved, and that the variations in burnup are within the maximum and minimum measured values.

CONCLUSIONS

A burnup importance function was defined in this paper as a weighting function to the local burnup change on the reactivity. With utilization of burnup importance function, physical features of the end effect were shown to be well understood in its application to the OECD/NEA/BUC Phase II-A and II-C models.

ACKNOWLEDGEMENT

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REFERENCES

1. J.C.Wagner and M.D. DeHart, "Review of Axial Burnup Distribution Considerations for Burnup Credit Calculations," *ORNL/TM*-1999/246 (2000).
2. Y. Naito, K. Suyama, F. Masukawa, K. Matsumoto, M. Kurosawa, and T. Kaneko, "Burnup Code for Fuel Assembly by Monte Carlo code, MKENO-BURN," *JAERI-Data/Code* 96-037(1996) [in Japanese].
3. R.E. Alcouffe, "DANTSYS: A Diffusion Accelerated Neutral Particle Transport Code System," *LA*-12969 (1996).
4. M. Takano and H. Okuno, "OECD/NEA Burnup Credit Criticality Benchmark - Result of Phase IIA -," *NEA/NSC/DOC*(96)01, *JAERI-Research* 96-003 (1996).
5. J.C. Neuber and T. Lamprect, "Phase II-C: Benchmark on the Effect of Axial Burnup Shapes," Revision 2, December 3, 2000; http://www.nea.fr/html/science/wpncs/buc/specifications/phase-2c.
6. A. Nouri, "OECD/NEA Burnup Credit Criticality Benchmark, Analysis of Phase II-B Results: Conceptual PWR Spent Fuel Transportation Cask," *Rapport IPSN*/98-05, *NEA/NSC/DOC*(98)1.
7. H. A. Tuan, H. Okuno and Y. Nomura, "Reactivity Effect of Axial Burnup Shapes for a Realistic PWR Spent Fuel Transport Cask," *Proceeding of the International Symposium NUCEF2001*, Japan Atomic Energy Research Institute (JAERI), Tokai, Ibaraki, Japan, October 31 – November 2, 2001, *JAERI-Conf* 2002-004, pp.399-406 (2002).

Table I. Parameters and case numbers (Indicated in **bold** type) adopted in Phase II-A benchmark

Cooling Time [y]	Fission Products	Burnup Profile	Mean Burnup [GWd/tU]		
			0	30	50
0	Yes	Yes	**1**	**2**	**3**
		No		**4**	N/A
	No	Yes		**5**	N/A
		No		N/A	N/A
1	Yes	Yes		**6**	**7**
		No		N/A	N/A
5	Yes	Yes		**8**	**9**
		No		N/A	N/A

Table II. Relative k_{eff} results to that of the average burnup shape (Case ID: 222222) for Phase II-C benchmark

Case ID*1	Relative k_{eff} *2 [%Δk/k]	Case ID*1	Relative k_{eff} *2 [%Δk/k]
111111	1.5	222333	-0.5
111112	1.5	223333	-1.0
111122	1.4	233333	-1.6
111222	1.1	333333	-2.0
112222	0.6	122223	0.3
122222	0.5	112233	0.6
222223	0.1	322221	-0.2
222233	-0.1	332211	-0.4

*1 ID represents burnup variation for the top 6 nodes; 1:min., 2: mean, 3: max.
*2 Relative to k_{eff} of Case ID 222222.

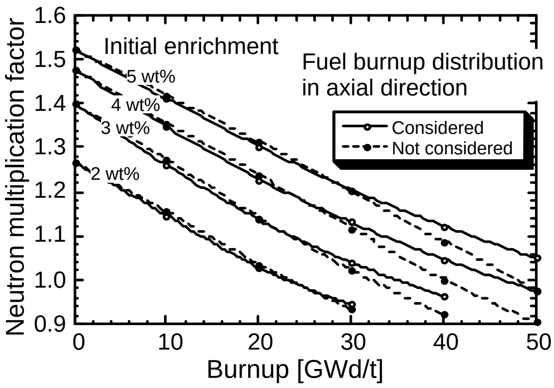


Figure 1. Effect of burnup profile with burnup and fuel enrichment.

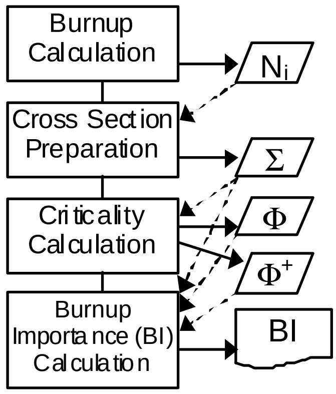
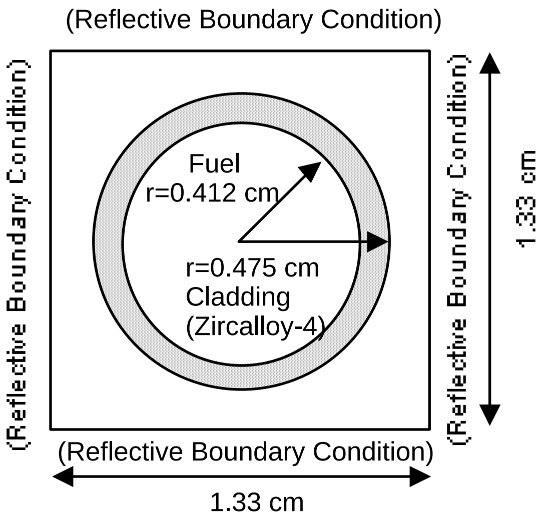
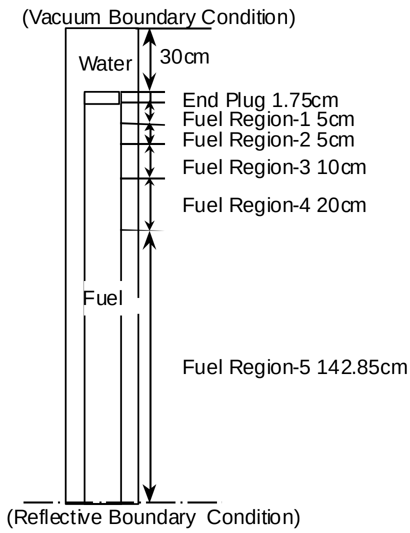


Figure 2. Calculation flow for burnup importance function.



(a) A horizontal view



(b) A vertical view

Figure 3. OECD/NEA/BUC Phase II-A model geometry.

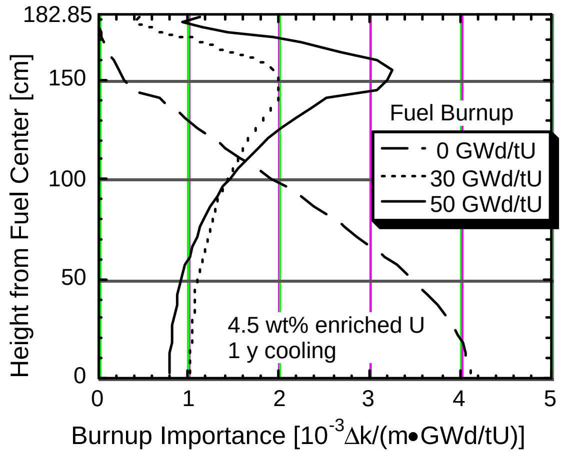


Figure 4. Burnup importance calculated for various burnups (1 y cooling).

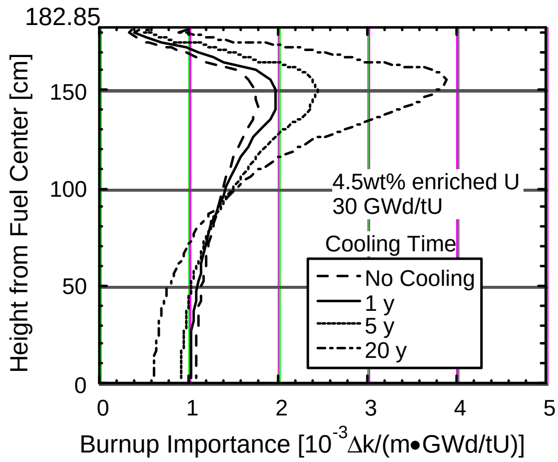


Figure 5. Burnup importance calculated for various cooling times (30 GWd/tU burnup).

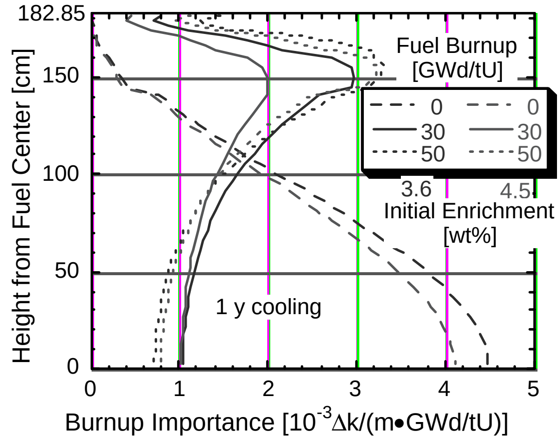


Figure 6. Burnup importance for the cases of the initial enrichment of 3.6 wt% compared with those of 4.5 wt%.

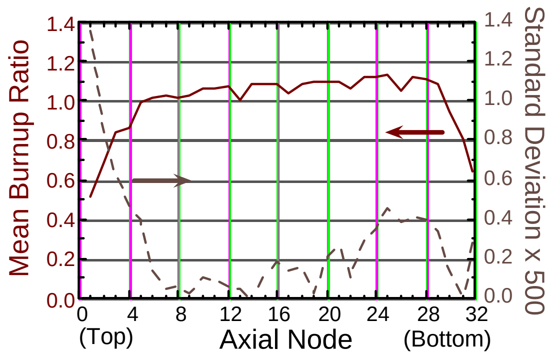


Figure 7. The mean and standard deviation of normalized axial burnup shapes in fuel assemblies received from the Nuclear Power Plant Neckarsheim II, as a function of the node number.

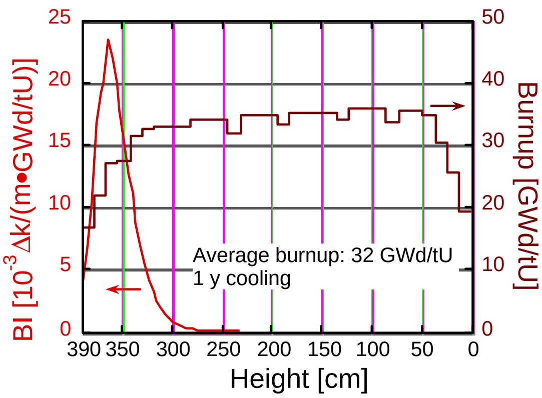


Figure 8. The burnup importance calculated for Phase II-C model in the mean burnup shape.