NEW ACCELERATION METHOD OF SOURCE CONVERGENCE FOR LOOSELY COUPLED MULTI UNIT SYSTEM BY USING MATRIX K CALCULATION

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ABSTRACT

In order to accelerate the slow convergence of the fission source distribution, the matrix k calculation has been developed and incorporated in the ordinary Monte Carlo calculation. The acceleration can be performed by the fission source correction with using the eigenvector of the fission source matrix equation, if the neutron coupling coefficients are approximately evaluated in the middle of Monte Carlo calculation where the fission source has not been converged yet. In this paper, we propose two effective applications of the matrix k calculation for the loosely coupled multi unit system, that is, the acceleration repetition method and the source generation method. The former simply repeats the acceleration procedure of matrix k calculation, and the calculated result for the irradiated fuel pin cell problem shows enough acceleration effect to obtain the reliable fission source on the statistical estimation for criticality. However, in some cases of the loosely coupled multi unit system, the repetition procedure of matrix k calculation more than twice could not be carried out to get into convergence because of many units of low source level. The latter is newly devised here in order to apply to such cases. The checkerboard fuel storage rack problem is one of the typical cases, and the calculated results show the effectiveness of this method.

1. INTRODUCTION

It is well known that the slow convergence of the fission source distribution for a loosely coupled system sometimes causes non-conservative estimation in the criticality safety analysis[1]. In order to investigate this problem and improve the robustness of criticality safety analyses, four different benchmark problems are proposed in the OECD/NEA expert group on source convergence. In this paper, we would like to describe the fission source acceleration methods by using the matrix k calculation in studying these benchmark problems. The matrix k calculation is one of the superior options to accelerate the slow convergence of fission source as it has been studied in the past[2][3][4]. Since the characteristic of slow convergence depends on the conditions of each problem such as the size of whole configuration, the number of fissile units, the reactivity differences and the intensity of neutron interactions between units, the acceleration effect of the matrix k calculation is apt to be different in each problem. In Chap.2, we briefly describe the conception of the acceleration method. Then the acceleration repetition method and the source generation method are proposed in Chap.3 and Chap.4, respectively. The calculation code and cross section library used in this paper are the continuous energy Monte Carlo code MCNP-4B2[5] and JENDL3.2[6].

2. CONCEPTION OF ACCELERATION BY USING MATRIX K CALCULATION

The fission source equation is described in the equation (1) that shows the neutron balance for the loosely coupled multi unit system (the number of units is n) in the state where the fission source distribution is converged at m'th cycle. In general, it can be derived from neutron transport theory. Here we briefly describe it in order to understand intuitively why the eigenvector of the fission source equation can accelerate the fission source convergence.

$$S_j^{m+1} = k_{eff} S_j^m = K_j S_j^m + \sum_{i \neq j}^{n} P_{ij} S_i^m = \sum_{i=1}^{n} P_{ij} S_i^m \tag{1}$$

Here,

- S_i, S_j: Fission source (number of neutron production points in the unit i and unit j)
- k_{eff}: Effective multiplication factor for the whole system
- P_{ij}: Coupling coefficient (probability that a fission neutron given rise in the unit i comes to the unit j and creates fission neutrons at the next cycle)
- K_i: Multiplication factor for the unit i (=P_{ii} : probability that a fission neutron given rise in the unit i and creates the fission neutrons in it at the next cycle)

The matrix expression for the equation (1) is described in the equation (2).

$$k_{eff} \mathrm{S} = \mathrm{PS} \tag{2}$$

Here,

- S: Fission source vector with elements S_i (i =1, n)
- P: Coupling coefficient matrix (or fission matrix) with elements P_{ij} (i, j = 1, n)

In the eigenvalue calculation based on the ordinary Monte Carlo (MC) method, criticality parameters (such as k_{eff}, and its standard deviation (1σ), flux, reaction rate, and so on) are statistically estimated after some skipped cycles starting from an initial guess of fission source distribution. The number of skipped cycles needs to be very large in order to converge the fission source distribution for the loosely coupled multi unit system. In this case, even though the unit-wise fission source levels are quite different from the converged one because of loose coupling among the units, the fission source distribution inside the unit becomes near the converged shape because the weak interactions among the units affect a little the fission source distribution inside the unit. In such a situation, the converged matrix elements P_{ij} could be approximately obtained by the fixed source calculation tracking neutrons from unit i to unit j based on the MC method where the neutron starting points are set to the fission source distribution inside the unit i on the previous eigenvalue calculation.

The conception of the acceleration methods proposed in this paper is based on the combination of the fixed source calculations to estimate P_{ij}'s and the successive matrix k calculation that solves deterministically the equation (2) to obtain the eigenvector of fission source matrix equation. Accordingly, the eigenvector is supposed to represent approximately converged unit-wise fission source levels. If the matrix k calculation is incorporated to the ordinary MC calculation by using eigenvector to correct the fission source distribution, an acceleration effect should appear on the slow convergence because the nearly converged eigenvector is approximately obtained regardless of the skipped cycles.

3. ACCELERATION REPETITION METHOD

3.1 PROCEDURE OF ACCELERATION REPETITION METHOD

As an effective application of the matrix k calculation, we propose a simple repetition of the acceleration procedure. Fig.1 shows the scheme of the acceleration repetition method. The procedure of this method is summarized below.

(1) The eigenvalue calculation by an ordinary MC with limited cycles is started from an initial guess (flat distribution may be generally tolerable). It creates the fission source distribution inside each unit.
(2) The fixed source MC calculation is performed starting from the neutron production points inside a certain unit i obtained in the previous step ((1) or (4)). The P_{ij}'s are simultaneously evaluated by tallying the neutron production rates in the range of the other units (unit j: j=1,n). The fixed source calculations as many times as the number of all units (unit i: i=1,n) give all P_{ij}'s.
(3) The fission source distribution of the previous step ((1) or (4)) is corrected by the eigenvector obtained by the matrix k calculation. When the acceleration is finished it jumps to the step (5).
(4) The eigenvalue MC calculation is restarted from the corrected fission source distribution to recreate the fission source distribution inside each unit. Then it goes back to the step (2) to repeat the acceleration procedure.
(5) Finally, the eigenvalue MC calculation is performed to estimate the criticality parameters.

How to determine the end of acceleration in the above procedure might be practically found after some acceleration repetitions performed.

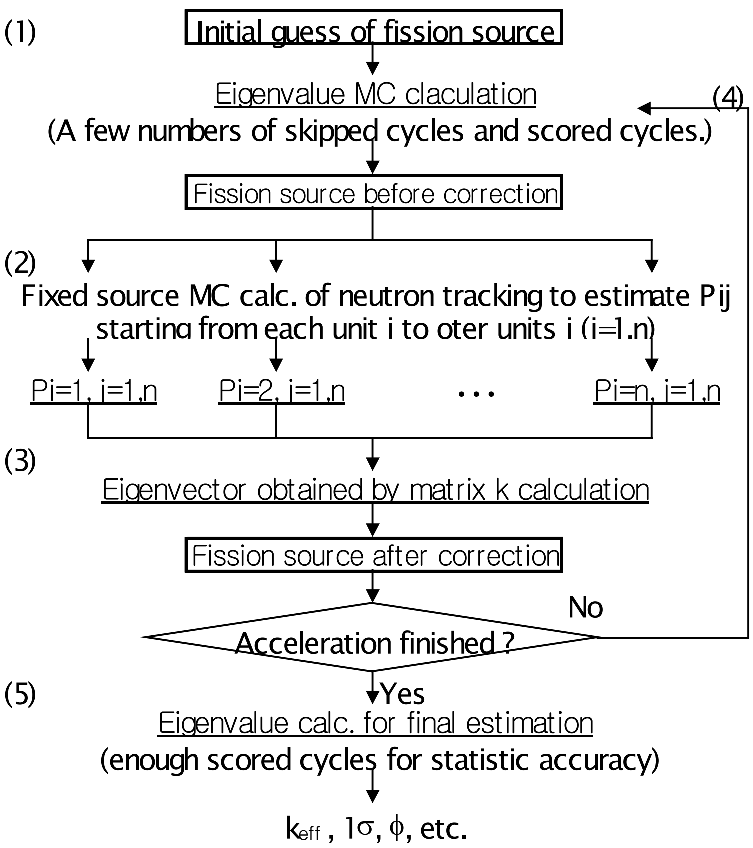


Figure 1. The scheme of the acceleration repetition method

3.2 SAMPLE CALCULATION OF ACCELERATION REPETITION METHOD

3.2.1 Configuration of Benchmark Problem No.2: Pin Cell Array with Irradiated Fuel

The configuration of OECD/NEA source convergence benchmark problem No.2 is modeled to be a radially infinite array of an irradiated fuel pin cell as shown in Fig.2. The water reflected boundary is considered at the top and the bottom of the pin cell array. The active fuel length is divided non-equidistantly into nine regions in the axial direction in order to model the realistic burnup distribution. OECD/NEA specifies the six cases of different burnup distribution. Here we treat one case shown in Table I, which is such a loosely coupled system that the very large region of the highest burnup at the middle of pin cell array makes the neutron coupling very weak between the upper region of the lowest burnup and the lower region of slightly higher burnup than the upper region.

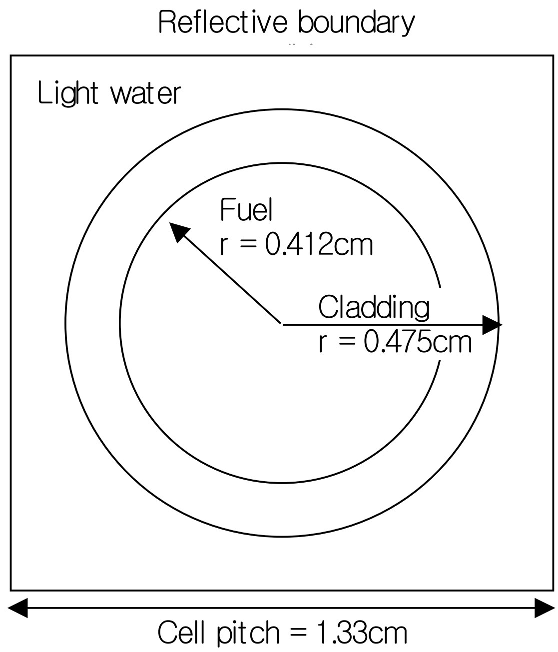


Figure 2. The radial configuration of the benchmark problem No.2 (The pin cell array with irradiated fuel)

Table I. The axial burnup distribution for the calculated case of the benchmark problem No.2

	Region height (cm)	Burnup (GWd/t)
Unit 1 (top)	5	21
Unit 2	5	24
Unit 3	10	30
Unit 4	20	40
Unit 5	285.7	55
Unit 6	20	55
Unit 7	10	40
Unit 8	5	30
Unit 9 (bottom)	5	24

3.2.2 Calculated Results

The acceleration procedure is repeated three times, in which the number of skipped cycles and the successive scored cycles for eigenvalue calculations (step (1) and (4) as described in the previous chapter) are set to be three and four, respectively. At the final estimation (step (5)), the number of skipped cycles is three and the number of scored cycles is two hundreds. The ordinary MC calculations of different calculation conditions are also performed in comparison with the acceleration repetition method. The number of histories per cycle is 10000 for all cases. The results of k_{eff} and its calculation time on our EWS machine for the various calculation conditions are shown in Table II. The comparison of k_{eff} trends per scored cycle among the typical cases is shown in Fig. 3. As shown in Table II, it is found that twenty is not enough number of the skipped cycles on the ordinary MC calculation for this problem such that the k_{eff} is underestimated in comparison with the reference calculation of large skipped cycles. As compared with this, the k_{eff} of the acceleration repetition method is almost same as the reference one. And it is almost constant from the beginning of the scored cycles as shown in Fig. 3.

Table II. The calculated results of k_{eff} for the benchmark problem No.2

Calculation conditions for the ordinary MC method		Results		
Skipped cycles	Scored cycles	k_{eff}	1σ	Time (min.)
20	200	1.05689	0.00052	235.72
40	200	1.05794	0.00046	239.13
55	200	1.05816	0.00045	250.96
70	200	1.05819	0.00045	284.14
Reference: 400	600	1.05810	0.00025	962.79
Acceleration repetition (3 times) & final estimation: (3skipped + 4scored) x 3, 3skipped + 200scored cycles		1.05802	0.00043	243.02

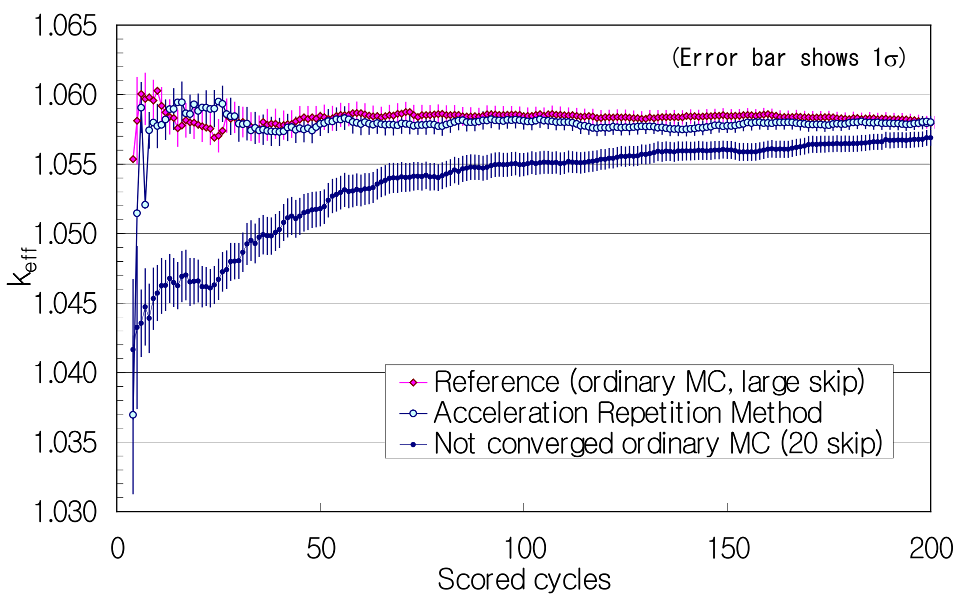


Figure 3. The comparison of keff trends per scored cycle for the benchmark problem No.2

The transformation of the fission source distribution in the acceleration repetition is shown in Fig. 4. As shown in Fig.4, it is found that the first acceleration procedure accelerates the unit-wise fission source drastically but does not reach enough convergence, and the unit-wise correction makes the fission source distribution discrete at the boundary between adjacent two units. Then, the fission source redistributes more smoothly to be nearly convergent during the three skipped cycles at the next step of the eigenvalue calculation. This makes the accuracy of the P_{ij} at the second acceleration procedure better than one at first. Thus the fission source distribution is accelerated. The number of three accelerations is enough and the further accelerations seem not to be effective because of low fission source levels that are nearly convergent. The acceleration repetition method is found effective in such case that the fission source redistribution inside the units can be distinctive in a few cycles as shown in this problem.

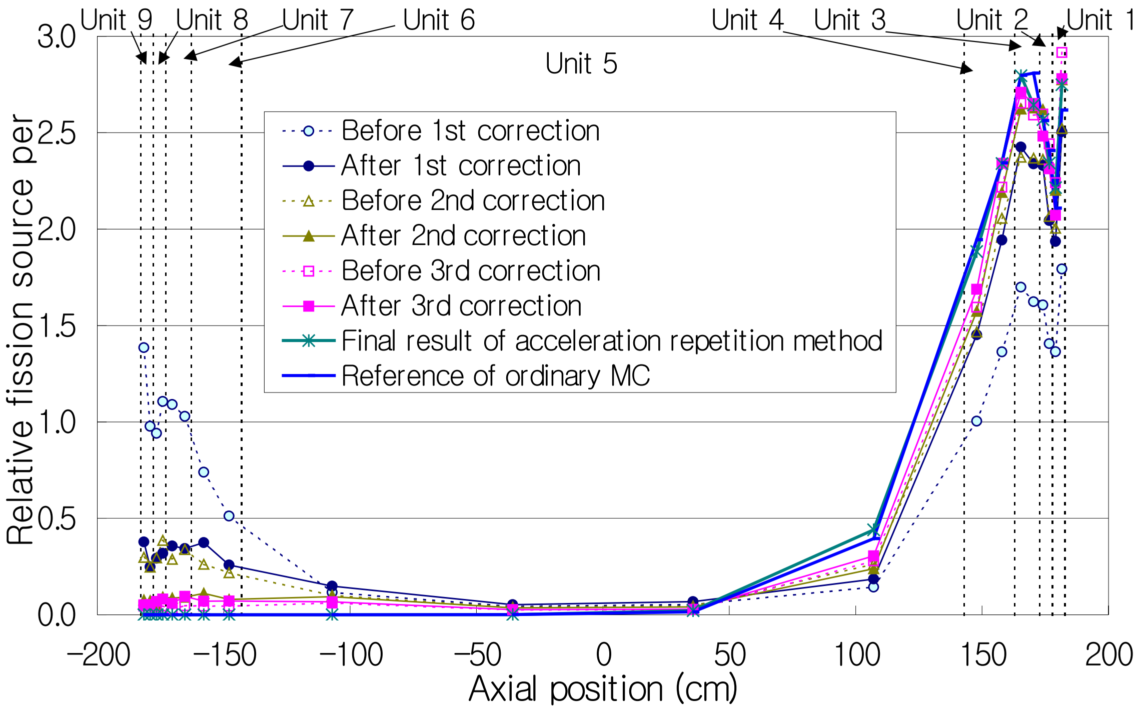


Figure 4. The transformation of fission source distribution

4. SOURCE GENERATION METHOD

4.1 PROCEDURE OF SOURCE GENERATION METHOD

In the acceleration repetition method, it may be expected that the more the acceleration repetitions are carried out, the more the P_{ij}'s are accurately estimated, because the fission source distribution inside the units obtained by the eigenvalue calculations would come to near convergence with the acceleration repeated. However, for a loosely coupled multi unit system, there are a lot of very low source levels located far from the peak one in the converged solution, especially in case where the reactivity of the particular unit is the greatest of all. That is because the small coupling coefficient makes the fission source level of the lower reactivity unit much lower than the one of the higher reactivity unit. Hence, the statistical errors of small P_{ij}'s for the units with very low source levels might sometimes yield anomalous results of the acceleration at the near convergence. One of the easy settlements for this statistical phenomenon is to increase the number of histories per cycle. But, in an extreme case, the acceleration repetition is impractical because some units have too low source levels to obtain accurate P_{ij}'s after the first acceleration even though the fission source distribution does not reach enough convergence or it costs too much calculation time to obtain the enough accuracy of the small P_{ij}'s because of the huge number of histories per cycle.

The source generation method proposed here as shown in Fig.5 is to generate the fission source distribution for a loosely coupled multi unit system in a realistic calculation time by just one matrix k calculation, where the P_{ij}'s are supposed to be estimated by the combinations of eigenvalue and fixed source calculations somewhat similar to the acceleration repetition method. In Fig.5, each of eigenvalue calculations starts from an initial guess of the neutron production points that are distributed only inside one unit i. It gives fission source distribution for the fixed source calculation to estimate the P_{ij}'s starting from the corresponding unit i to other units j. Although the unit-wise source levels may not reach convergence such that the fission source level for the unit i might be still maximal because of slow convergence, the distribution inside the unit i could be nearly converged because of the weak neutron interactions to the unit i from the other unit j. Therefore, the fission source inside the corresponding units for the fixed source calculations and the eigenvector obtained by the matrix k calculation adopting the above P_{ij}'s can produce the fission source distribution for the final estimation of criticality parameters.

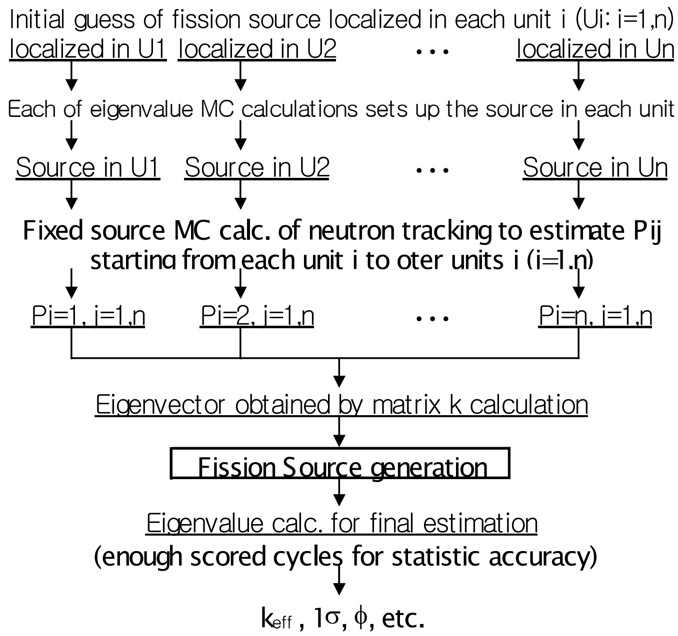


Figure 5. The scheme of the source generation method

4.2 SAMPLE CALCULATION OF SOURCE GENERATION METHOD

4.2.1 Configuration of Benchmark Problem No.1: Checkerboard Storage of Fuel Assemblies

The OECD/NEA source convergence benchmark problem No.1 is one of the typical cases for the loosely coupled multi unit system, and applicable for the source generation method. The configuration named the checkerboard storage here is shown in Fig.6. It is modeled on a notional 24x3 LWR fuel storage rack with thirty-six assemblies stored in alternate locations. The fuel assemblies are formed from a 15x15 square lattice of Zirconium-clad UO_2 fuel rods enriched to about 5.0%. They are centrally located within fully water-flooded steel storage racks surrounded by concrete on the three sides, water on the remaining side and water on the top and bottom. In the checkerboard storage, the water channels make the neutron interactions between assemblies very weak. In the vertical direction there is no axial variation in the main part of fuel elements.

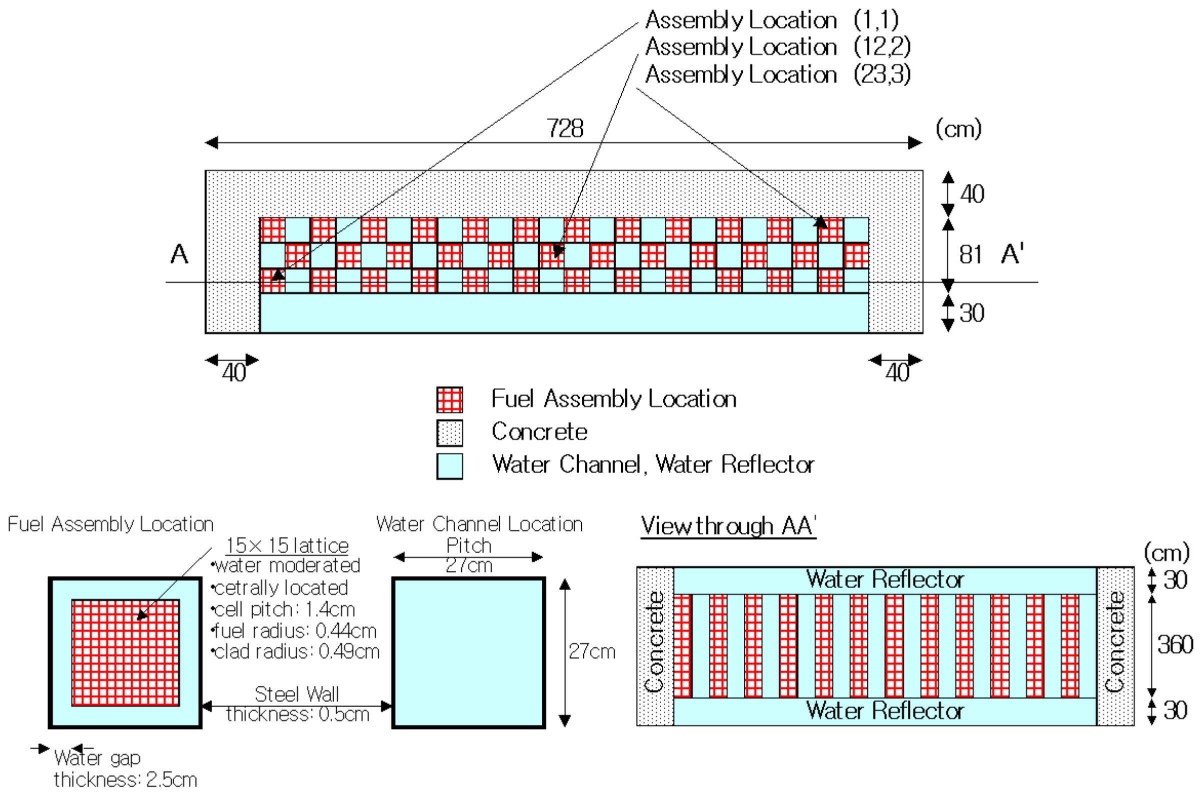
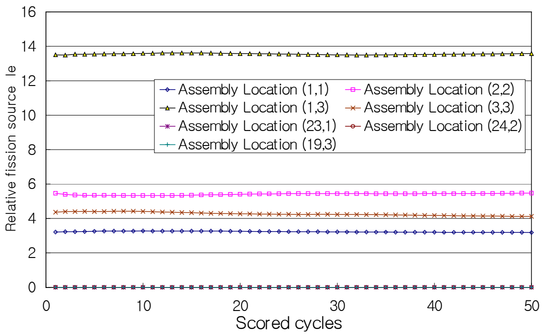


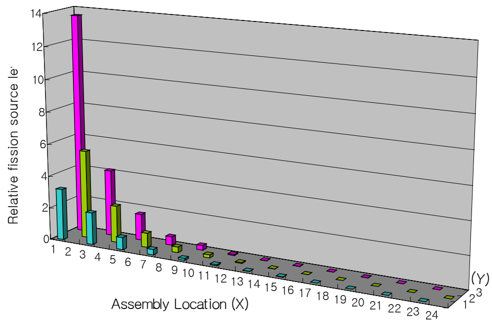
Figure 6. The configuration of the benchmark problem No.1
(The checkerboard storage of fuel assemblies)

4.2.2 Calculated Results

In the benchmark problem No.1, the calculation conditions for thirty-six cases are specified. The results that we have already calculated[7] show that all ordinary MC calculations in the specified conditions do not achieve the convergence such that the initial fission source distributions affect strongly the ones at the five-hundredth cycle. So, as shown in Ref.7, we have also estimated the converged fission source distribution with the specialized method named the source estimation method here. In the estimation method, a MC calculation to estimate finally the criticality parameters is started from the eigenvector of the fission source matrix equation which elements are estimated by eigenvalue MC calculations for three kinds of single assembly system to obtain the corresponding K_i's, and, four kinds of diagonally placed two assembly system and two kinds of multi assembly system to obtain the P_{ij}'s. The calculated results of the fission source trend per cycle and its distribution are shown in Fig.7 (1) and Fig.7 (2), respectively. As shown in Fig.7 (1), the fission source distribution is almost converged because of trending almost constantly. As shown in Fig.7 (2), the converged solution for the fission source distribution has the maximum on the assembly located at the upper-left corner where the K_i becomes the maximum because of the concrete boundary of the assembly adjacent on its two sides, and the fission source level decreases exponentially as far from the upper-left corner. There are many low source levels and the minimum of the assembly-wise fission fraction becomes less than 10^{-5}. Hence, it doesn't make the acceleration repetition method give a good result.



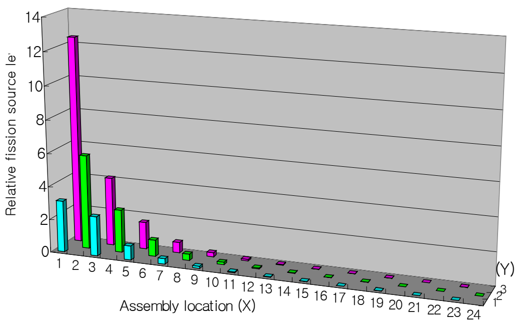
(1) The trends of fission source levels per scored cycle



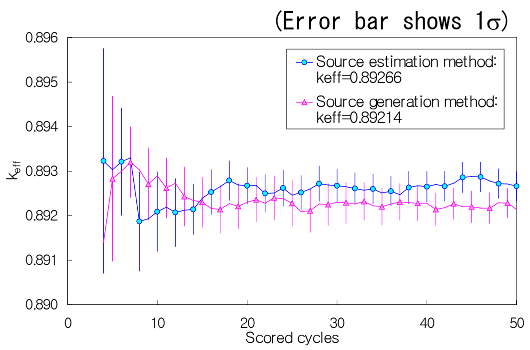
(2) The fission source distribution of the final estimation

Figure 7. The calculated results based on the source estimation method

The source estimation method described above gives a good result for the checkerboard storage problem, but it is generally inconvenience in some viewpoints of methodology. Cares must be taken how many kinds of isolated units are consisted of the system to estimate all K_i's. In some problems, every P_{ij}'s cannot be estimated for the combinations of adjacent two or multi unit system, or the number of cases may become too large. The source generation method described in the previous section is considered rather generalized procedure to obtain P_{ij}'s than the source estimation methods. The calculated results are shown in Fig.8 (1)-(2). Fig.8 (1) shows the eigenvector by the matrix k calculation of the source generation method. The k_{eff} trend in scored cycles is shown in Fig.8 (2) in comparison with the source estimation method. It found that the difference of P_{ij}'s between both methods makes the fission source distribution slightly different, therefore, it affects just a little on the k_{eff} trend. The source generation method also gives almost converged solution for the checkerboard storage problem.



(1) The eigenvector by matrix k calculation



(2) The k_{eff} trend of source generation method in comparison with source estimation method

Figure 8. The calculated results based on the source generation method

Finally, we discuss the comparison of calculation time among the different methods in order to achieve the reliable fission source distribution for statistically estimation. In Table III, we show the actual results for both of the source estimation method and the source generation method. The number of histories per cycle for statistically estimation is set to 100,000 in considering with the many low source levels suggested by the eigenvectors. So, in the ordinary MC method, the number of histories per cycle should be also set to 100,000, and the number of skipped cycle might be more than 1,000. The calculation time for the ordinary MC is inferred from this assumption. Table III shows the comparison of the number of total histories and the calculation time before statistically estimation among the three methods. The source estimation method can calculate the P_{ij}'s most efficiently by deciding the different kinds of units for the checkerboard storage problem. The source generation method takes about twice time in calculation, but it is still very efficient because of its advantage of short of our consideration time. In addition, it takes much less time than the ordinary MC.

Table III. The comparison of the number of the total histories and time to reach convergence

Method	Details	Number of total histories	Calculation time on our EWS
Ordinary MC	100,000 histories/cycle×1,000 cycles	100,000,000 (By inference)	10 days (By inference)
Source estimation	3 single units, 4 cases of adjacent 2-unit: (3+4)×2,000hst/cyc×(20skip+500active)cycles 2 cases of multi unit configuration: 2×5,000hst/cyc×(20skip+500active)cycles	12,480,000	1.5 days
Source generation	50,000hst/cyc×{Eigenvalue cal. (5skip+5active)+ Fixed source cal. (correspond to about 5cycles)}×36units	27,000,000	3 days

CONCLUSIONS

The acceleration repetition method and the source generation method by using the matrix k calculation are newly proposed in this paper. These methods are not always effective for all problems because the applicability of them would depend on not only calculation conditions but also the affinity to each problem. However, the fact that the both of the sample calculation give good results in the viewpoints of calculation time and accuracy suggests that the matrix k calculation can effectively accelerate the slow convergence of the fission source distribution on the MC calculation for the loosely coupled multi unit system.

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