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A New Generalized Multigroup Method

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ABSTRACT

Multigroup cross sections calculation has been dominated by the conventional multigroup method. In this method, a neutron flux spectrum in an infinite medium is assumed. By this assumption, the spectrum is rigidly fixed within each energy group. Hence, when the transport equation is solved, the neutron spectrum in each energy group is shifted to fit the solution. This causes the spectrum to be discontinuous. This approximation of the spectrum is unrealistic because the true spectrum is continuous.

To overcome these limitations, a new method that imposes continuity at the energy boundaries and allows variation of the spectrum within each energy group is being proposed. The new approach sheds the traditional definition of energy groups. The new method allows an energy to have partial membership in more than one group, which allows the assumed group spectral shape to adapt to problem-dependent conditions.

For a continuous and linearly varying spectrum, the energy membership functions are the "chapeau" or "hat" functions. These membership functions introduce new concepts such as group-to-group total cross sections, and up scattering in the fission and slowing-down regions. The added flexibility in the calculation of the group cross sections allows for more accurate solution to the transport equation. Preliminary results on a sample problem have demonstrated significant improvement in the solution when the same number of energy groups is utilized.

1. INTRODUCTION

In the multigroup transport equation, the subdivision of the continuous energy domain is approximated using a finite number of energy groups. The generation of group cross sections is predominantly performed using the traditional multigroup method (1). A new approach is presented that sheds the traditional definition of energy groups. The new method allows an energy to have partial membership in more than one group, which allows the assumed group spectral shape to adapt to problem-dependent conditions. It will be applicable in criticality safety studies because it allows for a more accurate fission neutron spectrum and spatial variation in the flux spectrum inside a homogenous material.

2. DISCUSSION

For an infinite homogeneous medium, with no fission and a constant neutron source, the transport equation is

$$\Sigma_t(E)\Phi(E) = \int_0^\infty dE' \Sigma_S(E' \to E)\Phi(E') + S(E) \tag{1}$$

To transform the equation into coupled multigroup equations, the flux Φ(E) is approximated by

$$\Phi(E) = \Psi(E) * \sum_{g=1}^{G} \phi_g f_g(E) \tag{2}$$

where Ψ(E) is an assumed spectrum shape, and f_g(E) is the group g membership function. By substituting Equation 2 into Equation 1, we obtain:

$$\Sigma_t(E)\Psi(E)\sum_g \phi_g f_g(E) = \int_0^\infty dE' \Sigma_s(E' \to E)\Psi(E')\sum_g \phi_g f_g(E') + S(E) \tag{3}$$

To obtain the coupled multigroup equations, we multiply this equation by each group membership function and integrate over energy for each group, which reduces to

$$\int_0^\infty \Sigma_t(E)\Psi(E)\sum_g \Phi_g f_g(E) f_k(E) dE = \int_0^\infty f_k(E) dE \int_0^\infty dE' \Sigma_s(E' \to E)\Psi(E')\sum_g \Phi_{g'} f_{g'}(E') + \int_0^\infty f_k(E) S(E) dE \tag{4}$$

For group k, for example, this gives us:

$$\sum_{g=1}^{G} \Phi_g \Sigma_{tkg} = \sum_{g=1}^{G} \Phi_g \Sigma_{skg} + S_k \tag{5}$$

where

$$\Sigma_{tgk} = \Sigma_{tkg} = \int_0^\infty \Sigma_t(E)\Psi(E) f_g(E) f_k(E) dE \tag{6}$$

$$\Sigma_{skg} = \int_0^\infty f_k(E) dE \int_0^\infty f_g(E') \Sigma_s(E' \to E)\Psi(E') dE' \tag{7}$$

$$S_k = \int_0^\infty S(E) f_k(E) dE \tag{8}$$

In the traditional multigroup method, $f_g(E)$ is a rectangular membership function with magnitude of $1/\Psi_g$ over the domain of group *g*, where the group cross sections are defined such that

$$\Psi_g \equiv \int_{E_g}^{E_{g-1}} dE \Psi(E) \tag{9}$$

$$\Sigma_{tgk} = \begin{cases} \dfrac{\int_{E_g}^{E_{g-1}} dE\, \Sigma_T(E)\Psi(E)}{\Psi_g}, & k = g \\ 0, & k \neq g \end{cases} \tag{10}$$

$$\Sigma_{sgg'} \equiv \frac{\int_{E_g}^{E_{g-1}} dE \int_{E_{g'}}^{E_{g'-1}} dE' \Sigma_s(E' \rightarrow E)\Psi(E')}{\Psi_{g'}} \tag{11}$$

For this research, we use the generalized form (Equations 5-8) and investigate the use of "chapeau" functions (shown in Figure 1) for the group membership functions.

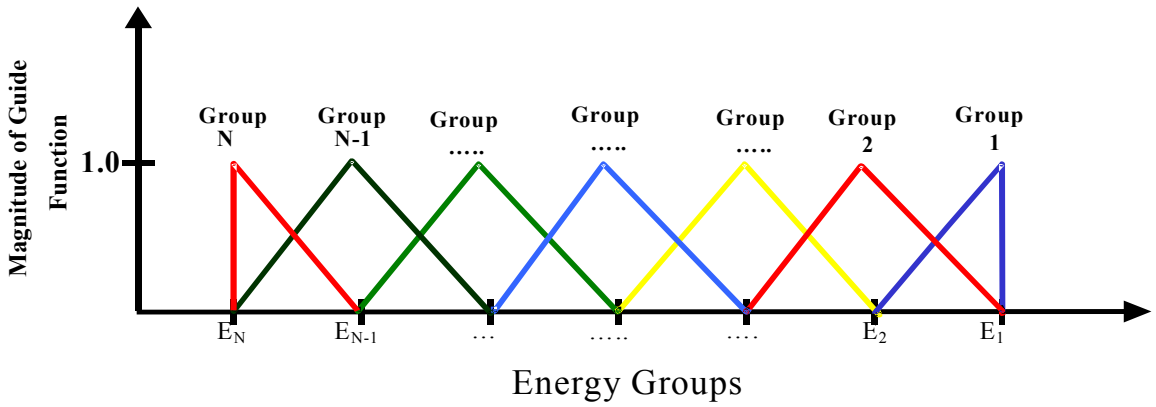

Figure 1: Chapeau Membership Functions for Generalized Multigroup

In the resulting multigroup equations, the group-to-group total cross sections are restricted to the current group and adjacent neighbors, e.g. with four groups the matrix relation would be:

$$\begin{bmatrix} \Sigma_{t11}-\Sigma_{s11} & \Sigma_{t21}-\Sigma_{s21} & 0 & 0 \\ \Sigma_{t12}-\Sigma_{s12} & \Sigma_{t22}-\Sigma_{s22} & \Sigma_{t32}-\Sigma_{s32} & 0 \\ -\Sigma_{s13} & \Sigma_{t23}-\Sigma_{s23} & \Sigma_{t33}-\Sigma_{s33} & \Sigma_{t34}-\Sigma_{s43} \\ -\Sigma_{s14} & -\Sigma_{s24} & \Sigma_{t34}-\Sigma_{s34} & \Sigma_{t44}-\Sigma_{s44} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{bmatrix} \tag{12}$$

3. EXAMPLE

To demonstrate the new method's robustness, an example problem was calculated. The problem consists of an infinite medium of ^{238}U with a constant neutron source emitting 1 neutron/second/unit volume with 500eV energy. The neutron slowing down rate at 400eV energy (calculated to be 0.22123 +/- 0.00013 using a Monte Carlo method) was calculated using both the traditional histogram and the chapeau group membership functions using a $1/(E*\Sigma_T)$ assumed spectrum shape. The number of groups was varied from 4 to 1024. This is a challenging problem because there is a 78% loss of neutrons over a very small energy range due the presence of absorption resonances and the small energy loss per collision.

4. RESULTS AND CONCLUSION

The resulting errors in the escape probabilities are shown in Figure 2. It is observed that the new method is generally more accurate than the traditional method. With 4 groups, the new method is within 7% of the true solution as opposed to the 28% for the traditional method. At 32 groups, the error for the new method is 0.8% compared to 3.35%. For 64 groups, both methods have converged to within the uncertainty of the true solution, which is about 0.1%. Hence the new method shows promise as an improved way or representing the energy dimension.

In the near future, this method is going to be applied to criticality safety, shielding, and reactor problems. The authors feel that the new method will have its greatest impact in criticality safety studies because it allows for a more accurate fission neutron spectrum (i.e., line segments vs. histogram) and spatial variation in the flux spectrum inside a homogenous material.

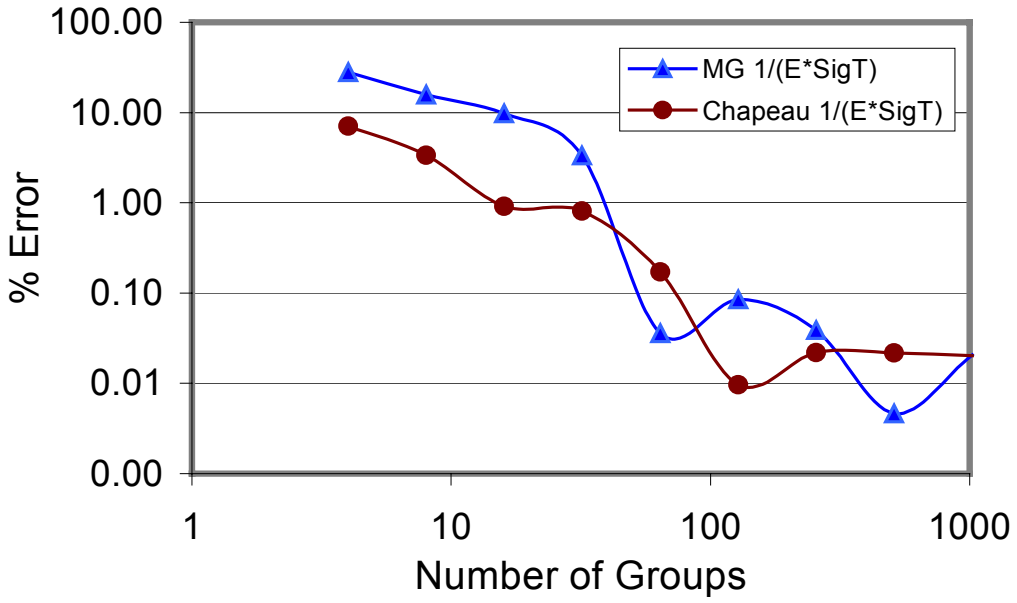

Figure 2: Error in the Estimation of Escape Probability vs. Number of Energy Groups for Different Methods of Calculating Group Cross Sections

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