SIMULATION OF ALL SPEED TWO-PHASE FLOWS

Moreira, M. L. and De Sampaio, P. A. B.
Instituto de Engenharia Nuclear/Comissão Nacional de Energia Nuclear (IEN/CNEN)
Av. Brigadeiro Trompowsky, S/N, Cidade Universitária, Ilha do Fundão
Caixa Postal: 68550, CEP 21945-970, Rio de Janeiro, RJ, Brasil
malu@ien.gov.br; sampaio@ien.gov.br

ABSTRACT

A new finite element formulation for compressible and nearly incompressible problems is applied to the analysis of two-phase flows. The method allows the analysis of nearly incompressible flows without resorting to the incompressible fluid model. Thus, the fluid equation of state is not discarded from the model. This feature is of foremost importance for the approach adopted in this work, where the thermodynamic behavior of the mixture is constructed from the thermodynamic behavior of the individual phases. Numerical examples of two-phase steam-water mixtures are presented. The models employed to characterize turbulence and the two-phase mixture are rather simple. Nevertheless, the examples presented show good qualitative behavior and serve to indicate further developments towards the computational simulation of two-phase flows. This paper extends the work of De Sampaio & Moreira [1] to deal with two-phase flows.

1. INTRODUCTION

Most numerical methods for compressible fluid dynamics present difficulties when applied to low speed (nearly incompressible) flows. The difficulties originate from the fast propagation of pressure waves, as flow conditions approach the incompressible limit. Nearly incompressible flows, i.e. flows characterized by very small Mach number, are usually approximated as fully incompressible. Thus, compressibility effects are eliminated at the modeling level, prior to considering any particular discretization method. In such a context, pressure is no longer a thermodynamic property related to density through a state equation. Most important, the pressure hyperbolic character and the associated wavelike propagation disappear from the model. In a fully incompressible flow, pressure takes an elliptic character: it must be determined from the momentum balance and boundary conditions, in such a way that a solenoidal velocity field is enforced. This paper presents a thermal hydraulic study of two-phase flow using a unified treatment for both compressible and nearly incompressible viscous flows. The computer program written to model two-phase flows employs a finite element formulation that has been demonstrated adequate for both compressible and nearly incompressible fluid dynamics. The robustness and versatility of the method has been demonstrated on steady and unsteady applications covering a wide range of Mach number [1]. The formulation is written combining conservative and non-conservative dependent variables, These are the mass-velocity vector (density*velocity), internal energy and pressure. Linear finite elements are used to approximate the field velocities. An important feature of the method is the implicit time discretization of the mass

balance and of the pressure terms appearing in the momentum and energy equations. Petrov-Galerkin weighting functions, derived from a least-squares procedure, are employed in the momentum and energy weighted residual statements. The resulting formulation automatically introduces *streamline upwinding* and pressure stabilizing terms. Moreover, the method retains stability, despite ignoring the short time-scales associated to the fast pressure transients of nearly incompressible flows. A segregated solution algorithm is employed. Once the pressure field in found, the scheme proceeds with the computation of the mass-velocity and internal energy fields. The cyclic update of pressure, mass velocity and internal energy requires the solution of symmetric systems of equations. This is accomplished using a preconditioning conjugate gradient solver. One of the most important features of the present formulation is that it allows the analysis of nearly incompressible flow problems without resorting to the incompressible model, and thus retaining the state equation of the fluid. In this paper, this characteristic is essential for the analysis of two-phase steam/water flows, where the thermodynamic of the mixture is modeled combining the thermodynamic behavior of the individual phases. This methodology has been tested for thermal equilibrium homogeneous two-phase flow model in simple two-dimensional configurations. In addition, in this work a local time-stepping algorithm is used in conjunction with a remeshing scheme. These improve the accuracy of the analysis.

2. CONSERVATION EQUATIONS FOR TWO-PHASE FLOW

The equations for conservation of mass, momentum and energy are defined on the open bounded domain Ω, with boundary Γ, contained in the *nsd*-dimensional space. The local volumetric mean equations of conservation of mass, momentum and energy are derived from the conservation equations of each phase [2], ignoring the superficial phenomena at the interface [3]. Thus, the governing equations can be written as follows:

$$\frac{\partial}{\partial t}\left(\alpha_1\ \rho_1 + \alpha_2\ \rho_2\right) + \nabla\cdot\left(\alpha_1\ \rho_1\ \mathbf{u}_1 + \alpha_2\ \rho_2\ \mathbf{u}_2\right) = 0 \tag{1}$$

$$\begin{aligned}&\frac{\partial}{\partial t}\left(\alpha_1\ \rho_1\ \mathbf{u}_1 + \alpha_2\ \rho_2\ \mathbf{u}_2\right) + \nabla\cdot\left(\alpha_1\ \rho_1\ \mathbf{u}_1\ \mathbf{u}_1 + \alpha_2\ \rho_2\ \mathbf{u}_2\ \mathbf{u}_2\right) + \\ &-\nabla\cdot\left[-\left(\alpha_1\ P_1 + \alpha_2\ P_2\right)\mathbf{I} + \left(\alpha_1\ \mathbf{S}_1 + \alpha_2\mathbf{S}_2\ \right)\right] - \left(\alpha_1\ \rho_1 + \alpha_2\ \rho_2\ \right)\mathbf{g} = \mathbf{0}\end{aligned} \tag{2}$$

$$\begin{aligned}&\frac{\partial}{\partial t}\left(\alpha_1\ \rho_1\ E_1 + \alpha_2\ \rho_2\ E_2\right) + \nabla\cdot\left(\alpha_1\ \ \rho_1\ \ \mathbf{u}_1\ H_1 + \alpha_2\ \rho_2\ \ \mathbf{u}_2\ H_2\right) + \\ &-\nabla\cdot\left(\alpha_1\ \ \mathbf{S}_1\cdot\ \mathbf{u}_1 + \alpha_2\ \mathbf{S}_2\cdot\mathbf{u}_2\right) + \nabla\cdot\left(\alpha_1\ \mathbf{q}_1 + \alpha_2\ \mathbf{q}_2\right) - \left(\alpha_1\ \rho_1\ \mathbf{u}_1 + \alpha_2\ \rho_2\ \mathbf{u}_2\ \right)\cdot\mathbf{g} = 0\end{aligned} \tag{3}$$

the subscripts "1" and "2" represent phases 1 e 2 respectively, α is the volumetric void fraction, ρ is the density, P is the thermodynamic pressure, $\mathbf{u}$ is the velocity vector, $\mathbf{I}$ is the identity tensor, $\mathbf{S}$ is the viscous stress, $\mathbf{g}$ is the gravitational field, E is the total specific energy, H is the total specific enthalpy, $\mathbf{q}$ is the heat flux and t is the time. The energy equation assumes that there is no energy source in the domain Ω.

The average two-phase mixture is modeled as $\rho = \alpha_1\ \rho_1 + \alpha_2\ \rho_2$; $\rho\ \mathbf{u} = \alpha_1\ \rho_1\ \mathbf{u_1} + \alpha_2\ \rho_2\ \mathbf{u_2}$; $\rho\ E = \alpha_1\ \rho_1\ E_1 + \alpha_2\ \rho_2\ E_2$; $P = \alpha_1\ P_1 + \alpha_2\ P_2$; $\mathbf{S} = \alpha_1\ \mathbf{S}_1 + \alpha_2\ \mathbf{S}_2$ and $\mathbf{q} = \alpha_1\ \mathbf{q}_1 + \alpha_2\ \mathbf{q}_2$. Using these definitions, the mixture equations can be written as follow [3]

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \, \mathbf{u}) = 0 \tag{4}$$

$$\frac{\partial}{\partial t}(\rho \, \mathbf{u}) + \nabla \cdot \left(\rho \, \mathbf{u} \, \mathbf{u} + \alpha_1 \, \alpha_2 \left(\frac{\rho_1 \, \rho_2}{\rho} \right) \mathbf{u}_r \, \mathbf{u}_r \right) - \nabla \cdot \left(- P \, \mathbf{I} + \mathbf{S} \right) - \rho \, \mathbf{g} = \mathbf{0} \tag{5}$$

$$\begin{aligned} &\frac{\partial}{\partial t}(\rho \, E) + \nabla \cdot \left(\rho \, \mathbf{u} \, H + \alpha_1 \, \alpha_2 \left(\frac{\rho_1 \, \rho_2}{\rho} \right) \mathbf{u}_r \, (\Delta \, H) \right) + \\ &- \nabla \cdot \left(\mathbf{S} \cdot \mathbf{u} + \frac{1}{\rho} \left[\alpha_1 \, \alpha_2 \, \rho_1 \, \mathbf{S}_2 - \alpha_1 \, \alpha_2 \, \rho_2 \, \mathbf{S}_1 \right] \cdot \mathbf{u}_r \right) + \nabla \cdot \mathbf{q} - \rho \, \mathbf{u} \cdot \mathbf{g} = 0 \end{aligned} \tag{6}$$

where $\mathbf{u}_r = \mathbf{u}_2 - \mathbf{u}_1$ and $H_k = E_K + \frac{P_K}{\rho_K} = \left(e_k + \frac{1}{2} \mathbf{u}_k \cdot \mathbf{u}_k \right) + \frac{P_K}{\rho_K} = h_k + \frac{1}{2} \mathbf{u}_k \cdot \mathbf{u}_k$.

The complete momentum and energy convection transport contains a term with the slip velocity $\mathbf{u}_r$. Nevertheless we omitted this term in our model as it is small compared with the main convection terms. Thus we used a no-slip model. Also, it is used the following approximation

$\Delta H = H_2 - H_1 \approx h_2 - h_1 = L$, where *L* is the latent heat of vaporization. So, for $\mathbf{u}_r = \mathbf{u}_2 - \mathbf{u}_1 = \mathbf{0}$, the conservation equations are written using the summation convention for *a =1,.. ,nsd* and *b=1,...,nsd*, in Cartesian co-ordinates as [3]:

$$\frac{\partial \rho}{\partial t} + \frac{\partial G_a}{\partial x_a} = 0 \tag{7}$$

$$\frac{\partial G_a}{\partial t} + u_b \frac{\partial G_a}{\partial x_b} + \frac{\partial u_b}{\partial x_b} G_a - \frac{\partial S_{ab}}{\partial x_b} + \frac{\partial P}{\partial x_a} - \rho g_a = 0 \tag{8}$$

$$\rho \left(\frac{\partial e}{\partial t} + u_b \frac{\partial e}{\partial x_b} \right) + P \frac{\partial u_b}{\partial x_b} + \frac{\partial q_b}{\partial x_b} - S_{ab} \frac{\partial u_a}{\partial x_b} = 0 \tag{9}$$

In the above equations G_a, u_a, P and e denote mass velocity (ρu_a), velocity, thermodynamic pressure and internal energy. The constitutive equations for viscous stress and heat flux are given by

$$S_{ab} = -\frac{2}{3} \mu \left(\frac{\partial u_c}{\partial x_c} \right) \delta_{ab} + \mu \left(\frac{\partial u_a}{\partial x_b} + \frac{\partial u_b}{\partial x_a} \right) \quad \text{and} \quad q_b = -\kappa \frac{\partial T}{\partial x_b}$$

where δ_{ab} is the Kronecker delta. The transport properties μ and κ are computed taking into account a volume ponderation of the properties of each phase and adding the turbulent viscosity and turbulent thermal conductivity estimated using the Smagorinsky model [5]. The state equation is given by

$$P = z(\rho, e) \, \rho \, e \tag{10}$$

Note that the above equations are written in dimensional form and density appears as dependent variable in the balance of mass. However, in order to derive a method suitable for a wide range of Mach number, we decided to introduce pressure as a main dependent variable instead. Thus the state equation is used to eliminate density from the mass balance, replacing it by pressure and internal energy. As we shall see, this will permit the derivation of a strongly implicit pressure equation, which is of foremost importance in regions of low Mach number. So, the governing equations can be written:

$$\alpha^* \frac{\partial p^*}{\partial t^*} - \beta^* \frac{\partial e^*}{\partial t^*} + \left(1 + \frac{\alpha^* \varphi^*}{Fr^2}\right) \frac{\partial G_a^*}{\partial x_a^*} = 0 \tag{11}$$

$$\frac{\partial G_a^*}{\partial t^*} + u_b^* \frac{\partial G_a^*}{\partial x_b^*} + \frac{\partial u_b^*}{\partial x_b^*} G_a^* - \frac{1}{Re} \frac{\partial S_{ab}^*}{\partial x_b^*} + \left(1 - \Phi_1\right) \frac{\partial p^*}{\partial x_a^*} - \frac{\Phi_1}{Fr^2} \rho^* g_a^* + \Phi_2 \frac{\partial e^*}{\partial x_a^*} = 0 \tag{12}$$

$$\rho^* \left(\frac{\partial e^*}{\partial t^*} + u_b^* \frac{\partial e^*}{\partial x_b^*} \right) + \left[z_0 + \gamma Ec \; p^* - \frac{\gamma Ec}{Fr^2} \rho^* \varphi^* \right] \frac{\partial u_b^*}{\partial x_b^*} + \frac{\gamma}{Re \; Pr} \frac{\partial q_b^*}{\partial x_b^*} - \frac{\gamma Ec}{Re} S_{ab}^* \frac{\partial u_a^*}{\partial x_b^*} = 0 \tag{13}$$

Note that using the mass balance and the state equation, the energy equation (13) can be written as

$$\begin{aligned} &\left[\rho^* + z \; \beta^* \left(e^* + 1\right)\right] \left(\frac{\partial e^*}{\partial t^*} + u_b^* \frac{\partial e^*}{\partial x_b^*} \right) + \frac{\gamma}{Re \; Pr} \frac{\partial q_b^*}{\partial x_b^*} - \frac{\gamma Ec}{Re} S_{ab}^* \frac{\partial u_a^*}{\partial x_b^*} \quad = \\ &\left[z \; \alpha^* \left(e^* + 1\right)\right] \left[\frac{\partial p^*}{\partial t^*} + \left(1 - \Phi_1\right) u_b^* \frac{\partial p^*}{\partial x_b^*} + \left(\frac{1 - \Phi_1}{Fr^2} \right) G_b^* g_b^* + \Phi_2 \; u_b^* \frac{\partial e^*}{\partial x_b^*} + \frac{\varphi^*}{Fr^2} \frac{\partial G_b^*}{\partial x_b^*} \right] \end{aligned} \tag{14}$$

The non-dimensional variables, denoted by the superscript '*', are related to the original dimensional variables according to

$$\begin{array}{llll} u_a = u_0 u_a^* & p' = P_0 + \rho_0 u_0^2 \; p'^* & T = T_0 \left(T^* + 1\right) & G_b = \rho_0 u_0 G_b^* \\ e = e_0 \left(e^* + 1\right) & \rho = \rho_0 \rho^* & \varphi = |\mathbf{g}| \; L \varphi^* & g_a = |\mathbf{g}| \; g_a^* \\ \mu = \mu_0 \; \mu^* & \kappa = \kappa_0 \; \kappa^* & x_a = L \; x_a^* & t = L \, t^* / u_0 \\ \alpha = \alpha^* / u_0^2 & \beta = \beta^* \rho_0 / e_0 & & \end{array}$$

where the subscript '$_0$' indicates reference values and L is the reference length. In the above conservation equations, we have also used the following definitions:

$$p' = P + \rho\varphi \quad ; \quad \varphi = -\mathbf{g} \cdot \mathbf{x} = -g_b \; x_b \quad ; \quad \alpha = \left(\frac{\partial \rho}{\partial P} \right)_e \quad ; \quad \beta = -\left(\frac{\partial \rho}{\partial e} \right)_P ; \; \gamma = \frac{T_0 \; c_p}{e_0} \quad ;$$

$$z_0 = \frac{P_0}{\rho_0 \; e_0} \quad ; \quad \Phi_1 = \frac{\alpha^* \varphi^*}{Fr^2 + \alpha^* \varphi^*} \quad ; \quad \Phi_2 = \frac{\beta^* \varphi^*}{Fr^2 + \alpha^* \varphi^*} \quad ;$$

$$S_{ab}^* = -\frac{2}{3} \mu^* \left(\frac{\partial u_c^*}{\partial x_c^*} \right) \delta_{ab} + \mu^* \left(\frac{\partial u_a^*}{\partial x_b^*} + \frac{\partial u_b^*}{\partial x_a^*} \right) \quad \text{and} \quad q_b^* = -\kappa^* \frac{\partial T^*}{\partial x_b^*} \; .$$

where the non-dimensional groups of Reynolds, Froude, Prandtl and Eckert are given by

$$Re = \frac{\rho_0\, u_0\, L}{\mu_0} \quad ; \quad Fr = \frac{u_0}{\sqrt{|\mathbf{g}|\, L}} \quad ; \quad Pr = \frac{\mu_0\, c_p}{\kappa_0} \quad \text{e} \quad Ec = \frac{u_0^2}{c_p\, T_0}.$$

In the remainder of this work we shall deal exclusively with the non-dimensional equations and the superscript '*' - used to indicate non-dimensional quantities - will be dropped. Also, the variable p' will be denoted by p.

3. THE DISCRETE FLUID FLOW MODEL

De Sampaio & Moreira [1] have presented the detailed methodology used in this work. Only the most relevant aspects will be treated here. Linear Lagrangian finite elements are employed to represent the mass-velocity, pressure and internal energy fields. The Galerkin method is used to obtain the discrete pressure equation, whilst a Petrov-Galerkin / Least-Squares based approach is used in the derivation of the discrete equations for mass-velocity and internal energy. The problem is solved using a segregated solution procedure. Once the pressure field is found, the algorithm proceeds with the computation of the mass-velocity and internal energy fields. The cyclic update of pressure, mass-velocity and internal energy requires the solution of symmetric systems of equations. This is accomplished with preconditioned conjugate gradient solvers, suitable for parallel and vector implementation on supercomputers.

3.1 PRESSURE EQUATION

Time implicit scheme for mass:

$$\alpha^n\left(\frac{p^{n+1}-p^n}{\Delta t}\right)-\frac{\beta^n}{\rho^{n+1/2}}\left[\rho^{n+1/2}\left(\frac{e^{n+1}-e'}{\Delta t}\right)\right]+\frac{\partial G_a^{n+1}}{\partial x_a}=\frac{\beta^n}{\rho^{n+1/2}}\left[\rho^{n+1/2}\left(\frac{e'-e^n}{\Delta t}\right)\right]-\frac{\alpha^n\varphi}{Fr^2}\frac{\partial G_a^n}{\partial x_a} \tag{15}$$

Time implicit splitting schemes based on the energy equation:

$$\rho^{n+1/2}\left(\frac{e'-e^n}{\Delta t}+\theta_1\, u_b^n\frac{\partial e'}{\partial x_b}+\theta_2\, u_b^n\frac{\partial e^n}{\partial x_b}\right)+\frac{\gamma}{Re\, Pr}\frac{\partial q_b^n}{\partial x_b}-\frac{\gamma Ec}{Re}S_{ab}^n\frac{\partial u_a^n}{\partial x_b}=0 \tag{16}$$

$$\rho^{n+1/2}\left(\frac{e^{n+1}-e'}{\Delta t}\right)+\left(z_0+\gamma Ec\, p^{n+1}-\frac{\gamma Ec}{Fr^2}\rho^{n+1/2}\varphi\right)\frac{\partial u_b^n}{\partial x_b}=0 \tag{17}$$

In the above equations, the superscripts n and $n+1$ denote the time-level and Δt is the time-step. Here, the parameters θ_1 and $\theta_2 = 1-\theta_1$ control the implicitness in the time discretization of the convective term (explicit for $\theta_1 = 0$ and implicit for $\theta_1 = 1$). When splitting the energy balance into equations (16) and (17), we have isolated the term representing the compressible contribution in Eq. (17). Equation (16), on the other hand, retains the remaining terms, typical of incompressible applications.

The substitution of equation (17) into equation (15) and using the Galerkin method we have a strongly implicit equation for pressure, where the momentum balance is used to write G_a^{n+1} in terms of the various forces acting on the fluid. In particular, the pressure gradient contribution in the momentum balance is approximated with an implicit time discretization. The final pressure equation involves pressure terms arising from the mass, momentum and energy (compressible part) equations. Most importantly, these pressure terms are approximated using a fully implicit time discretization, which introduces numerical damping of pressure errors. This permits retaining stability in the pressure computation, despite ignoring the short time-scales associated to the fast pressure waves that characterize nearly incompressible flows. The final discrete pressure equation can be written in matricial form as

$$\mathbf{A} \quad \hat{p}^{n+1} = \quad \mathbf{B} \quad \hat{p}^{n} \quad + \quad \mathbf{C} \quad , \quad \forall \; p_i^{n+1} \quad \text{free} \tag{18}$$

where the matrix **A** has the important mathematical properties of symmetry and positive definiteness and the matrices **B** and **C** contain the source terms [1].

It is important to note that before Eq. (18) can be solved, one has to determine the internal energy field $\hat{e}'$ corresponding to the solution of Eq. (16). This task, which can be regarded as a pre-processing for the pressure update. The matricial form of the internal energy equation (16) obtained from a Petrov-Galerkin / Least-Squares based approach can be written as

$$\mathbf{D} \quad \hat{e}' \quad = \quad \mathbf{E} \quad \hat{e}^{n} \quad = \quad \mathbf{F} \quad , \; \forall \; e' \quad \text{free} \tag{19}$$

As in the pressure equation, the matrix **D** has the important mathematical properties of symmetry and positive definiteness and the matrices **E** and **F** contain the source terms [1].

3.2 MASS VELOCITY AND INTERNAL ENERGY EQUATIONS

Once the internal energy $\hat{e}'$ and the pressure $\hat{p}^{n+1}$ are determined, the next stage is the computation of the new mass-velocity $\hat{G}_a^{n+1}$ and new internal energy $\hat{e}^{n+1}$. Again, Petrov-Galerkin weighted residual statements are employed in the discretization, for the update of the mass velocity $\hat{G}_a^{n+1}$ and the internal energy $\hat{e}^{n+1}$. The discrete equations can be written in matricial form as

$$\mathbf{J} \quad \hat{G}_a^{n+1} \quad = \quad \mathbf{M} \quad \hat{G}_a^{n} \quad + \quad \mathbf{N} \quad , \qquad \forall \; G_{ai}^{n+1} \quad \text{free} \tag{20}$$

$$\mathbf{P} \quad \hat{e}^{n+1} \quad = \quad \mathbf{Q} \quad \hat{e}^{n} \quad + \quad \mathbf{R} \quad , \quad \forall \; e_i^{n+1} \quad \text{free} \tag{21}$$

where the matrices **J** and **P** are symmetric are positive definite. The matrices **M, N, Q** and **R** contain source terms.

4. NUMERICAL EXAMPLES

The formulation presented in the previous sections has been applied to the analysis of two types of problems involving phase change. The first example shows the phase change due to the compressibility effects, whilst the second example shows the phase change due to the addition of heat.

4.1. FLOW BETWEEN TWO HORIZONTAL PLATES

It is presented the vapor flow between two horizontal plates. Initially, there is stagnated saturated vapor. At instant *t=0,* vapor in the same thermodynamic conditions is injected at the inlet section (left side of the domain). Adiabatic and no-slip boundary conditions are applied on the solid walls. The initial thermodynamics conditions and the prescribed velocity at the inlet are used to define the Reynolds and Eckert number of the analysis (*Re=182, Ec=1,81*).
We observe that the vapor injection causes a pressure transient and condensation of vapor occurs in the compressed region. Figures 1 e 2 present the evolution of this transient. The phenomenon is strongly compressible and supersonic conditions are attained.

4.2. FLOW BETWEEN TWO VERTICAL PLATES WITH HEATING

Initially, subcooled water flows up between two parallel plates at ΔT_{sat} =1°K. No-slip boundary conditions are applied to the solid walls. The Reynolds e Eckert number for the analysis are based on the initial thermodynamics and velocity conditions (Re=801, Ec=4,93 10^{-9}). A wall heat flux is applied on the vertical walls and a heating process with phase change is started. Figure 3 shows that at the beginning of the transient nucleate boiling occurs only next to the walls, while the major volume of the fluid is subcooled. At the end of the transient, almost the totality of the fluid is saturated and it is observed bulk boiling.

CONCLUSIONS

The above numerical examples have demonstrated the versatility of the proposed method on dealing with phase changes and covering a wide range of Mach number. For the simulation of two-phase flow, the methodology showed a good qualitative behavior. Extensions for two-phase flows with slip model are planned to be implemented in the near future. This will give us results that are more realistic. It is worth stressing that the numerical method presented needs a more sophisticated model for the computation of turbulent flows.

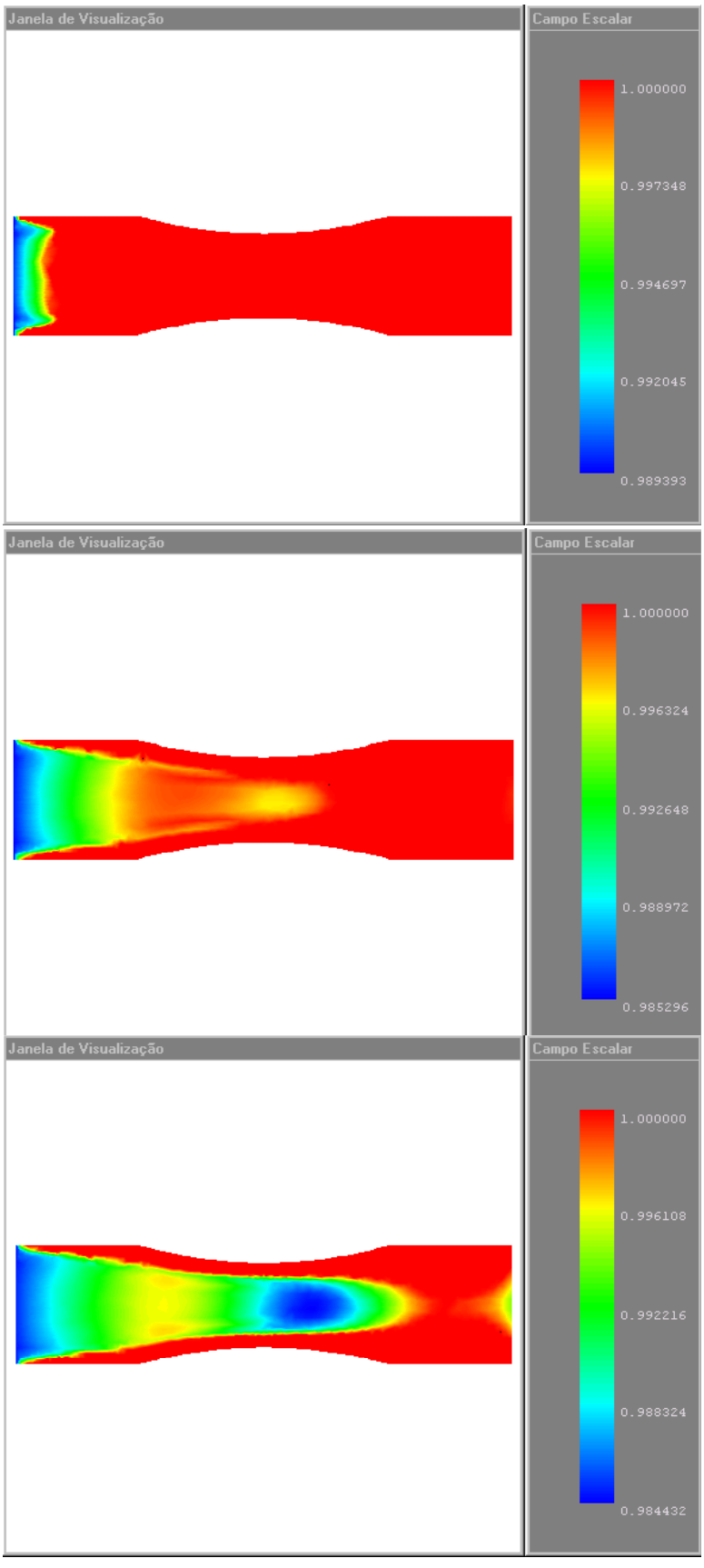

Figure 1. Vapor mass fraction field during the transient.

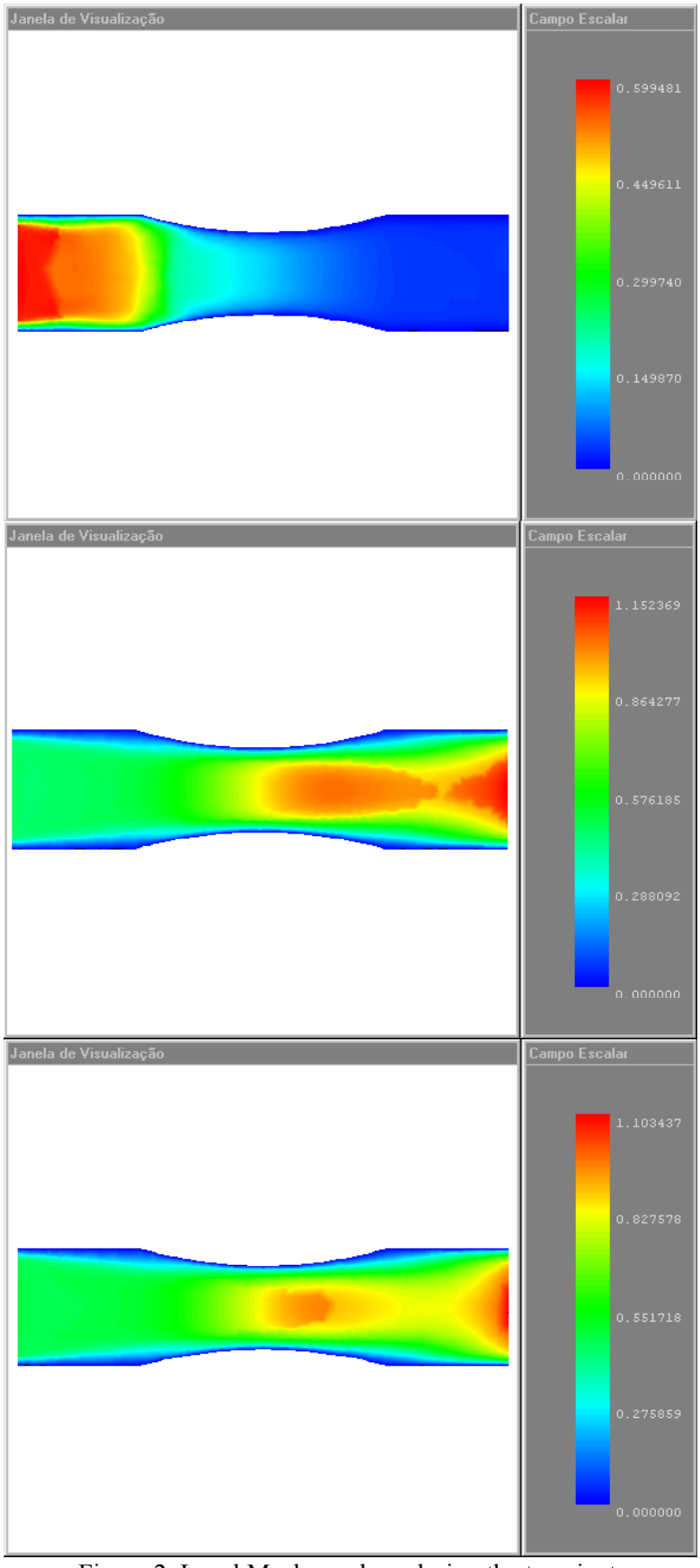

Figure 2. Local Mach numbers during the transient.

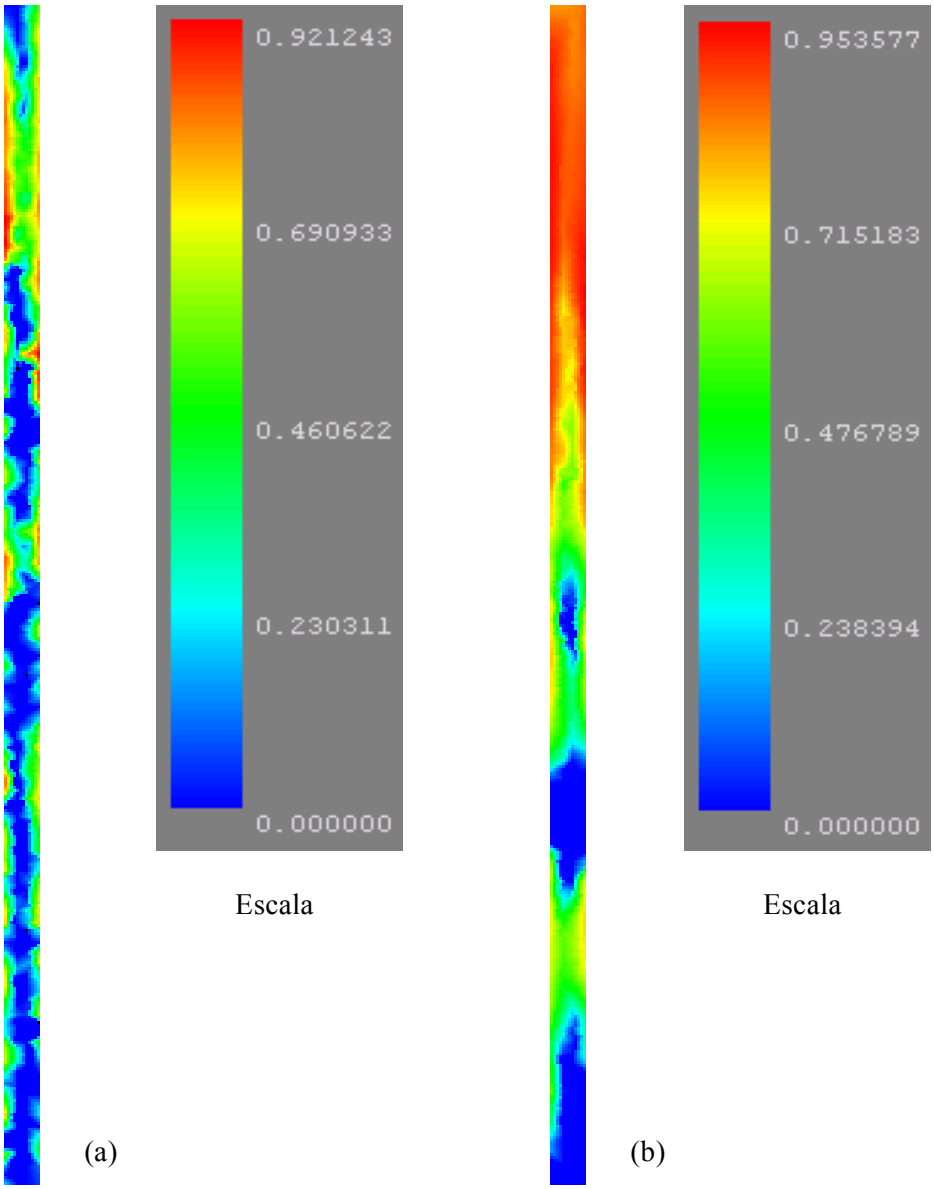

Figure 3. Void fraction field: (a) middle of the transient; (b) end of the transient.

REFERENCES

1. P.A.B. De Sampaio, & M. L. Moreira, "A new finite element formulation for both compressible and nearly incompressible fluid dynamics", *International Journal for Numerical Methods in Fluids*, **vol. 32**, pp. 51-78 (2000).
2. J. M. Delhaye, M. Giot, & M. L. Riethmuller, *Thermohydraulics of Two-Phase Systems for Industrial Design and Nuclear Engineering*, Hemisphere Publishing (1981).
3. M. L. Moreira, "Simulação computacional de escoamentos viscosos compressíveis / quase incompressíveis", *D.Sc. thesis*, Programa de Engenharia Civil, COPPE/UFRJ, BRASIL, 1998.
4. N. S. Lapa, "Um novo algoritmo computacional para análise termohidráulica de plantas industriais", *M.Sc. thesis*, Programa de Engenharia Nuclear, COPPE/UFRJ, BRASIL (1998).
5. M. Lesieur, *Turbulence of Fluids*, Academic Publishers, Holland (1990).