RESULTS OF A C5G7 TEST COMPUTED USING WIMS-SH+SHM-SQADRE COMPLEX

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ABSTRACT

The results of a C5G7 test computed using Surface Harmonics Method and Surface Pseudosources Method are presented in the paper.

1. INTRODUCTION

Paper [1] proposed a problem with 2-D and 3-D assemblies formed of PWR 17x17 fuel-assemblies as a test one. The assembly includes both uranium and graded MOX fuel-assemblies.

Responding to the challenge, we have carried out our calculations of this assembly using the Surface Harmonics Method [2] under various approaches within the method. Computations have been made within two stages. On the first stage, we prepared effective cell characteristics. The second stage was devoted to the assembly calculation itself. During the cell computation we were calculating symmetric, asymmetric and complementary trial functions. For symmetric and asymmetric trial functions we applied Surface Pseudo-Sources Method (SPSM) [2]. This method is based on the use of Green's function of the infinite media for transport equation in cylindrical geometry. Symmetric trial functions, which are determined absorption and fission cross sections mainly, were obtained in G_3 – approximation of SPSM but asymmetric trial functions necessary for diffusion coefficients were obtained in G_2 approximation [3]. We have applied WIMS-SH code for calculations of all above mentioned functions.

For calculating less important trial functions we used diffusion approximation for homogenized cells.

In case of 3F SHM approximation, we used 3 trial functions per cell. The corresponding SHM equation may be written in the form

$$\frac{2}{a^2}\sum_{j=1}^{4}\hat{D}_0\left(\hat{D}_j+\hat{D}_0\right)\hat{D}_j(\vec{\mathbf{F}}_0-\vec{\mathbf{F}}_j)+\hat{\mathbf{S}}\vec{\mathbf{F}}_0-\frac{1}{\lambda}\nu_f\hat{\mathbf{S}}_f=0 \qquad (1)$$

In case of 4F approximation we have the following system of two 5-point equations:

$$\frac{2}{a^2}\sum_{j=1}^{4}\hat{D}_0\left(\hat{D}_j+\hat{D}_0\right)\hat{D}_j(\vec{\mathbf{F}}_0-\vec{\mathbf{F}}_j)+\hat{\mathbf{S}}\vec{\mathbf{F}}_0-\frac{1}{\lambda}\nu_f\hat{\mathbf{S}}_f+\left[\frac{2}{a^2}\sum_{j=1}^{4}\hat{D}_0\left(\hat{D}_j+\hat{D}_0\right)\hat{D}_j(\vec{X}_0-\vec{X}_j)\cos\alpha_j\right]=0$$
$$\frac{2}{a^2}\sum_{j=1}^{4}\hat{D}_0\left(\hat{D}_j+\hat{D}_0\right)\hat{D}_j(\vec{X}_0-\vec{X}_j)\cos^2\alpha_j-\hat{\mathbf{S}}_X\vec{\mathbf{F}}_0+\left[\frac{2}{a^2}\sum_{j=1}^{4}\hat{D}_0\left(\hat{D}_j+\hat{D}_0\right)\hat{D}_j(\vec{\mathbf{F}}_0-\vec{\mathbf{F}}_j)\cos\alpha_j\right]=0 \quad (2)$$

Here $\hat{D}_j, \hat{\mathbf{S}}, \hat{\mathbf{S}}_X, \nu_f\hat{\mathbf{S}}_f$ - non-diagonal matrixes, including matrixes of effective diffusion coefficients,

a - lattice mesh; summing up within equations is carried out using indexes of neighbor cells,

α_j - angle between orts to the boundaries j and 0.

For solving this problem within active zone, we have applied SHM-SQARE code [4].

Corresponding equations look similar to finite-difference diffusion equations but have quite different meaning of the variables. The latter are some combinations of matrixes of cell response type boundary level/incoming current. We made no assumptions about internal structure of cells when deriving the method and, in this respect, the SHM may be considered as a straightforward attack on transport equations. 3D calculation was carried out using mesh size 3.88 cm in height (56 slabs). High collapsed results are in Tables 9-11.

2. RESUTS AND DISSCUSIONS

The type of equations we used justifies quite small CPU-time costs associated with problem solving (see Table 1). We have used PC-Celeron-667, 64 Mb RAM Windows-98 for that purpose.

Table 1 CPU-time, PC-Celeron-667, 64 Mbt-RAM

Dimension	2D-problem		3D-problem
Approximation of SHM	3F	4F	3F
CPU-time	20 c	25 c	40 m

K_{eff} values obtained for various approximations are presented in the Table 2.

Table 2. Eigenvalues

Dimension	2D-problem		3D-problem
Approximation of SHM	3F	4F	3F
Eigenvalues	1.18569	1.18589	1.18323

Table 3. Power distribution end errors %, for <3F> approximation SHM. UO2-1 Fuel assembly, 2D-problem.

2.218 -0.9	2.224 -0.9	2.234 -0.9	2.246 -1.0	2.253 -0.9	2.255 -1.2	2.208 -1.1	2.170 -1.0	2.142 -1.0	2.077 -1.1	2.020 -1.1	1.971 -1.3	1.876 -1.3	1.773 -1.3	1.650 -1.2	1.500 -1.3	1.292 -0.8
	2.236 -1.1	2.263 -1.0	2.303 -1.3	2.330 -1.2	2.400 -1.3	2.279 -1.1	2.237 -1.0	2.276 -1.2	2.141 -1.1	2.084 -1.0	2.098 -1.2	1.942 -1.3	1.820 -1.3	1.674 -1.2	1.509 -1.1	1.292 -0.9
		2.338 -1.1	2.477 -1.4	2.503 -1.2	0.000 0.0	2.416 -1.4	2.366 -1.3	0.000 0.0	2.267 -1.2	2.207 -1.4	0.000 0.0	2.088 -1.4	1.959 -1.5	1.733 -1.3	1.530 -1.3	1.297 -1.0
			0.000 0.0	2.532 -1.4	2.493 -1.6	2.323 -1.2	2.271 -1.2	2.322 -1.2	2.174 -1.1	2.122 -1.1	2.176 -1.5	2.107 -1.4	0.000 0.0	1.841 -1.5	1.561 -1.3	1.304 -0.9
				2.432 -1.4	2.463 -1.5	2.306 -1.2	2.257 -1.2	2.308 -1.4	2.161 -1.1	2.108 -1.1	2.152 -1.4	2.027 -1.4	2.001 -1.4	1.858 -1.5	1.581 -1.2	1.310 -1.0
					0.000 0.0	2.379 -1.5	2.332 -1.4	0.000 0.0	2.234 -1.3	2.176 -1.2	0.000 0.0	2.056 -1.4	1.970 -1.5	0.000 0.0	1.633 -1.2	1.315 -1.3
						2.243 -1.2	2.201 -1.3	2.254 -1.3	2.109 -1.2	2.054 -1.3	2.084 -1.4	1.927 -1.4	1.838 -1.3	1.796 -1.6	1.550 -1.3	1.288 -0.9
							2.162 -1.1	2.214 -1.3	2.073 -1.2	2.018 -1.2	2.045 -1.4	1.889 -1.3	1.800 -1.3	1.762 -1.6	1.525 -1.3	1.268 -1.0
								0.000 0.0	2.123 -1.3	2.066 -1.4	0.000 0.0	1.935 -1.6	1.843 -1.6	0.000 0.0	1.558 -1.3	1.258 -1.1
									1.988 -1.1	1.936 -1.1	1.964 -1.3	1.815 -1.2	1.731 -1.4	1.696 -1.4	1.468 -1.2	1.222 -0.8
										1.887 -1.2	1.915 -1.2	1.774 -1.3	1.695 -1.4	1.658 -1.6	1.435 -1.2	1.194 -0.7
											0.000 0.0	1.816 -1.4	1.743 -1.5	0.000 0.0	1.455 -1.2	1.174 -0.7
												1.713 -1.4	1.696 -1.5	1.586 -1.4	1.354 -1.1	1.126 -0.8
													0.000 0.0	1.499 -1.4	1.278 -1.4	1.076 -0.9
														1.335 -1.3	1.188 -1.2	1.020 -0.6
															1.090 -0.8	0.958 -0.4
																0.878 0.0

2.379 -1.5	Power for SHM calculation Error, % [Ref-SHM]*100./Ref

Table 4. Power distribution end errors %, for <3F> approximation SHM. PU Fuel assembly , 2D-problem.

1.314 -0.2	1.057 0.4	0.934 0.3	0.864 0.0	0.814 0.1	0.773 -0.5	0.713 0.0	0.662 0.4	0.622 -0.1	0.568 0.5	0.521 1.1	0.482 0.6	0.432 1.4	0.390 2.0	0.368 2.7	0.397 3.6	0.594 1.3
1.297 -0.1	1.334 0.7	1.162 0.7	1.092 0.2	1.039 1.0	1.049 -0.2	0.909 1.2	0.843 1.1	0.841 -0.2	0.723 1.4	0.662 1.5	0.653 0.7	0.555 2.3	0.494 2.1	0.455 3.3	0.500 3.4	0.586 1.1
1.291 -0.2	1.310 0.7	1.165 0.9	1.181 -0.5	1.116 0.4	0.000 0.0	0.957 -0.9	0.874 -0.3	0.000 0.0	0.759 -0.4	0.688 0.0	0.000 0.0	0.600 1.2	0.536 1.3	0.458 3.9	0.491 3.4	0.582 1.0
1.293 -0.1	1.324 0.8	1.266 -0.3	0.000 0.0	1.109 0.5	1.138 -0.9	0.944 1.5	0.866 1.7	0.878 0.1	0.744 2.0	0.683 2.1	0.701 0.2	0.577 2.3	0.000 0.0	0.503 2.4	0.497 3.2	0.582 1.1
1.296 -0.2	1.341 1.2	1.284 0.3	1.174 0.4	1.123 1.6	1.089 -0.5	0.913 1.4	0.841 1.8	0.856 -0.1	0.722 2.1	0.662 2.1	0.674 0.3	0.591 3.3	0.525 2.7	0.503 3.6	0.503 3.6	0.582 0.8
1.300 -0.5	1.419 0.3	0.000 0.0	1.289 -0.5	1.146 -0.3	0.000 0.0	0.990 -0.7	0.904 -0.2	0.000 0.0	0.787 -0.2	0.712 0.1	0.000 0.0	0.615 0.8	0.570 1.5	0.000 0.0	0.535 2.4	0.583 1.0
1.274 -0.1	1.314 1.4	1.227 -0.2	1.127 1.6	1.021 1.7	1.042 -0.4	0.883 1.8	0.818 1.7	0.836 -0.2	0.704 2.6	0.643 2.6	0.650 0.6	0.544 3.2	0.503 3.7	0.477 2.4	0.494 3.8	0.574 1.0
1.255 -0.1	1.293 1.4	1.202 -0.3	1.104 1.7	1.001 1.7	1.022 -0.4	0.870 2.0	0.807 1.8	0.823 0.0	0.695 2.3	0.635 2.6	0.640 0.7	0.535 3.0	0.494 3.9	0.469 2.4	0.488 4.3	0.569 1.1
1.245 -0.2	1.357 0.1	0.000 0.0	1.192 -0.4	1.077 -0.4	0.000 0.0	0.948 -0.5	0.870 0.1	0.000 0.0	0.758 -0.1	0.686 0.6	0.000 0.0	0.585 0.9	0.531 1.9	0.000 0.0	0.517 2.7	0.567 0.9
1.210 0.0	1.250 1.4	1.166 -0.3	1.070 1.8	0.971 1.7	0.995 -0.5	0.847 1.9	0.785 2.0	0.804 0.0	0.678 2.6	0.620 2.7	0.626 0.6	0.523 3.3	0.483 3.7	0.460 2.3	0.478 3.8	0.556 1.3
1.184 0.1	1.224 1.5	1.143 0.0	1.055 1.6	0.957 1.8	0.978 -0.6	0.833 1.8	0.772 2.1	0.789 0.1	0.667 2.2	0.612 2.7	0.618 0.8	0.518 3.2	0.480 3.8	0.455 2.8	0.472 4.1	0.550 1.5
1.166 0.0	1.279 0.7	0.000 0.0	1.167 -0.6	1.044 0.0	0.000 0.0	0.907 -0.2	0.833 -0.1	0.000 0.0	0.727 0.0	0.660 0.9	0.000 0.0	0.573 1.3	0.531 1.7	0.000 0.0	0.500 3.3	0.546 1.5
1.120 0.0	1.168 1.5	1.127 0.6	1.031 1.1	0.992 2.3	0.969 0.0	0.815 2.2	0.754 2.0	0.772 0.0	0.653 2.7	0.601 2.8	0.614 1.3	0.538 3.9	0.478 3.5	0.461 3.4	0.462 4.1	0.534 1.4
1.077 -0.2	1.114 0.9	1.078 -0.1	0.000 0.0	0.954 1.1	0.985 0.0	0.822 1.9	0.758 2.2	0.772 0.3	0.657 2.4	0.607 3.0	0.624 1.4	0.516 3.1	0.000 0.0	0.453 2.9	0.448 4.1	0.524 1.7
1.034 0.0	1.070 1.2	0.970 1.3	0.996 0.4	0.948 1.3	0.000 0.0	0.823 0.2	0.758 0.3	0.000 0.0	0.664 0.6	0.607 1.1	0.000 0.0	0.533 2.5	0.478 2.3	0.410 4.7	0.439 4.2	0.518 1.7
1.003 0.2	1.079 0.9	0.972 1.3	0.931 1.3	0.898 1.9	0.914 1.0	0.802 2.1	0.749 2.2	0.751 1.2	0.652 2.5	0.601 2.8	0.595 2.1	0.508 3.8	0.455 3.6	0.421 4.3	0.459 4.0	0.526 2.2
1.011 0.4	0.904 0.6	0.851 0.4	0.815 0.6	0.783 0.6	0.753 0.4	0.704 0.8	0.661 0.9	0.625 0.9	0.576 1.4	0.533 1.6	0.495 2.1	0.448 2.7	0.408 3.0	0.384 3.2	0.407 3.7	0.568 1.8

Table 5. Power distribution end errors %, for <3F> approximation SHM. UO2-2 Fuel assembly, 2D-problem.

0.792 0.5	0.791 0.0	0.774 -0.1	0.752 -0.1	0.727 0.0	0.701 0.0	0.658 0.5	0.619 0.6	0.586 0.4	0.541 0.8	0.501 1.1	0.465 1.5	0.422 2.1	0.386 2.3	0.365 3.1	0.381 2.7	0.496 1.4
	0.830 -0.4	0.836 -0.5	0.831 -0.4	0.815 -0.3	0.811 -0.4	0.741 -0.1	0.697 -0.1	0.679 0.1	0.610 0.5	0.565 1.2	0.542 1.1	0.479 1.5	0.435 1.9	0.405 2.2	0.416 1.9	0.524 1.1
		0.869 -0.4	0.906 -0.5	0.891 -0.4	0.000 0.0	0.804 -0.5	0.755 -0.2	0.000 0.0	0.662 0.3	0.614 0.7	0.000 0.0	0.530 1.4	0.481 2.0	0.431 2.1	0.431 1.5	0.531 1.0
			0.000 0.0	0.895 -0.5	0.855 -0.4	0.768 -0.1	0.721 0.1	0.707 0.1	0.633 0.7	0.589 0.9	0.576 1.0	0.533 1.6	0.000 0.0	0.456 2.0	0.436 1.6	0.527 0.9
				0.841 -0.2	0.826 -0.3	0.747 0.4	0.703 0.6	0.689 0.4	0.618 0.8	0.575 1.3	0.559 1.7	0.504 2.0	0.483 2.3	0.452 2.4	0.433 2.2	0.516 0.7
					0.000 0.0	0.750 0.4	0.707 0.7	0.000 0.0	0.623 0.9	0.579 1.1	0.000 0.0	0.500 1.9	0.464 2.1	0.000 0.0	0.435 2.4	0.502 1.5
						0.684 0.8	0.646 1.0	0.636 1.1	0.571 1.4	0.530 1.4	0.514 1.9	0.455 2.3	0.420 2.6	0.414 2.7	0.402 2.4	0.478 1.5
0.613 1.3	Power for SHM calculation Error, % [Ref-SHM]*100./Ref						0.613 1.3	0.603 1.1	0.541 1.8	0.503 1.8	0.486 2.3	0.430 2.6	0.397 3.3	0.392 3.0	0.381 2.9	0.454 1.6
								0.000 0.0	0.534 1.7	0.495 2.2	0.000 0.0	0.425 2.5	0.391 3.2	0.000 0.0	0.374 2.9	0.432 1.6
									0.480 1.8	0.446 2.3	0.432 2.7	0.382 2.8	0.354 3.1	0.349 3.5	0.340 3.0	0.403 1.9
										0.415 2.6	0.402 2.7	0.357 3.3	0.330 3.7	0.326 3.1	0.317 3.2	0.376 2.0
											0.000 0.0	0.349 3.0	0.324 3.2	0.000 0.0	0.305 3.3	0.351 2.3
												0.314 3.6	0.301 3.6	0.284 3.6	0.271 3.8	0.320 2.6
													0.000 0.0	0.259 4.2	0.246 4.2	0.292 3.0
														0.230 5.3	0.226 4.8	0.268 3.3
															0.222 4.1	0.257 3.2
																0.278 2.9

Table 6. Power distribution end errors %, for <4F> approximation SHM. UO2-1 Fuel assembly, 2D-problem.

2.217 -0.9	2.222 -0.8	2.232 -0.8	2.243 -0.8	2.252 -0.9	2.247 -0.9	2.209 -1.1	2.171 -1.1	2.135 -0.7	2.077 -1.1	2.021 -1.2	1.964 -0.9	1.875 -1.2	1.770 -1.1	1.648 -1.1	1.497 -1.1	1.284 -0.2
	2.235 -1.0	2.262 -1.0	2.297 -1.0	2.335 -1.4	2.393 -1.0	2.286 -1.4	2.243 -1.3	2.268 -0.9	2.147 -1.4	2.089 -1.3	2.091 -0.9	1.947 -1.6	1.815 -1.1	1.672 -1.0	1.506 -0.9	1.285 -0.3
		2.338 -1.1	2.464 -0.8	2.503 -1.2	0.000 0.0	2.403 -0.8	2.353 -0.8	0.000 0.0	2.254 -0.6	2.195 -0.8	0.000 0.0	2.087 -1.4	1.949 -1.0	1.734 -1.3	1.527 -1.1	1.289 -0.4
			0.000 0.0	2.535 -1.5	2.475 -0.9	2.329 -1.5	2.279 -1.6	2.309 -0.7	2.181 -1.4	2.129 -1.4	2.161 -0.8	2.110 -1.5	0.000 0.0	1.830 -1.0	1.555 -0.9	1.296 -0.3
				2.448 -2.0	2.451 -1.0	2.314 -1.6	2.266 -1.6	2.296 -0.8	2.169 -1.5	2.116 -1.5	2.142 -0.9	2.040 -2.0	2.004 -1.6	1.859 -1.5	1.583 -1.4	1.303 -0.5
					0.000 0.0	2.367 -1.0	2.319 -0.9	0.000 0.0	2.223 -0.8	2.165 -0.7	0.000 0.0	2.047 -0.9	1.956 -0.8	0.000 0.0	1.627 -0.8	1.303 -0.4
						2.252 -1.6	2.210 -1.7	2.242 -0.8	2.117 -1.6	2.061 -1.7	2.073 -0.9	1.934 -1.7	1.843 -1.6	1.786 -1.1	1.554 -1.6	1.282 -0.4
0.613 1.3	Power for SHM calculation Error, % [Ref-SHM]*100./Ref						2.171 -1.5	2.202 -0.8	2.080 -1.6	2.026 -1.6	2.034 -0.9	1.896 -1.7	1.807 -1.7	1.752 -1.1	1.528 -1.5	1.263 -0.6
								0.000 0.0	2.112 -0.7	2.056 -0.9	0.000 0.0	1.925 -1.1	1.833 -1.1	0.000 0.0	1.551 -0.8	1.247 -0.2
									1.996 -1.5	1.944 -1.5	1.954 -0.8	1.821 -1.6	1.737 -1.7	1.686 -0.8	1.471 -1.4	1.217 -0.4
										1.895 -1.6	1.906 -0.7	1.782 -1.7	1.700 -1.7	1.649 -1.0	1.438 -1.4	1.190 -0.4
											0.000 0.0	1.808 -0.9	1.732 -0.8	0.000 0.0	1.449 -0.7	1.164 0.1
												1.725 -2.1	1.699 -1.6	1.585 -1.4	1.356 -1.3	1.120 -0.3
													0.000 0.0	1.491 -0.8	1.274 -1.0	1.069 -0.2
														1.335 -1.3	1.187 -1.1	1.014 0.0
															1.089 -0.7	0.954 0.1
																0.874 0.5

Table 7. Power distribution end errors %, for <4F> approximation SHM. PU Fuel assembly, 2D-problem.

1.319 -0.5	1.057 0.4	0.930 0.8	0.859 0.5	0.811 0.5	0.765 0.5	0.712 0.2	0.662 0.4	0.616 0.9	0.567 0.7	0.520 1.3	0.477 1.7	0.431 1.6	0.389 2.3	0.365 3.3	0.398 3.3	0.604 -0.3
1.303 -0.6	1.338 0.4	1.164 0.6	1.089 0.5	1.050 0.0	1.046 0.1	0.920 0.0	0.851 0.0	0.837 0.3	0.732 0.2	0.669 0.5	0.651 1.0	0.561 1.3	0.493 2.3	0.456 3.1	0.503 2.8	0.597 -0.7
1.296 -0.6	1.313 0.5	1.169 0.5	1.169 0.6	1.119 0.2	0.000 0.0	0.946 0.3	0.863 1.0	0.000 0.0	0.749 0.9	0.680 1.2	0.000 0.0	0.600 1.2	0.530 2.4	0.460 3.4	0.493 3.0	0.592 -0.7
1.297 -0.4	1.323 0.9	1.252 0.9	0.000 0.0	1.114 0.0	1.119 0.8	0.956 0.3	0.879 0.2	0.867 1.3	0.756 0.4	0.693 0.6	0.691 1.6	0.582 1.5	0.000 0.0	0.498 3.3	0.498 3.0	0.591 -0.4
1.301 -0.5	1.354 0.2	1.283 0.4	1.182 -0.2	1.148 -0.6	1.074 0.9	0.927 -0.1	0.852 0.4	0.842 1.6	0.734 0.5	0.672 0.7	0.665 1.6	0.605 1.0	0.528 2.1	0.503 3.6	0.510 2.3	0.592 -0.9
1.299 -0.4	1.416 0.5	0.000 0.0	1.268 1.1	1.130 1.1	0.000 0.0	0.975 0.8	0.890 1.3	0.000 0.0	0.775 1.4	0.701 1.6	0.000 0.0	0.606 2.1	0.560 3.3	0.000 0.0	0.535 2.4	0.591 -0.3
1.280 -0.6	1.328 0.4	1.210 1.2	1.143 0.3	1.036 0.2	1.026 1.1	0.899 0.0	0.831 0.2	0.823 1.4	0.716 0.9	0.654 1.0	0.640 2.2	0.553 1.7	0.510 2.4	0.471 3.6	0.501 2.5	0.585 -0.9
1.261 -0.6	1.306 0.4	1.185 1.1	1.119 0.3	1.015 0.3	1.006 1.2	0.885 0.4	0.819 0.4	0.811 1.4	0.707 0.6	0.646 1.0	0.630 2.2	0.544 1.4	0.501 2.5	0.462 3.9	0.494 3.1	0.579 -0.6
1.244 -0.1	1.351 0.5	0.000 0.0	1.178 0.8	1.059 1.3	0.000 0.0	0.933 1.1	0.856 1.7	0.000 0.0	0.747 1.4	0.675 2.2	0.000 0.0	0.576 2.4	0.524 3.2	0.000 0.0	0.516 2.8	0.575 -0.4
1.217 -0.6	1.262 0.4	1.149 1.1	1.086 0.3	0.985 0.2	0.979 1.1	0.860 0.3	0.798 0.4	0.791 1.6	0.690 0.9	0.630 1.1	0.616 2.2	0.532 1.7	0.490 2.3	0.453 3.8	0.484 2.6	0.567 -0.6
1.190 -0.4	1.237 0.5	1.128 1.3	1.070 0.2	0.971 0.4	0.963 1.0	0.848 0.1	0.785 0.4	0.777 1.6	0.679 0.4	0.621 1.1	0.608 2.2	0.527 1.6	0.486 2.6	0.449 4.0	0.478 2.9	0.560 -0.3
1.164 0.2	1.277 0.8	0.000 0.0	1.148 1.0	1.029 1.4	0.000 0.0	0.893 1.3	0.819 1.5	0.000 0.0	0.716 1.5	0.650 2.4	0.000 0.0	0.565 2.7	0.522 3.4	0.000 0.0	0.501 3.1	0.553 0.2
1.124 -0.3	1.180 0.5	1.126 0.7	1.038 0.5	1.014 0.1	0.956 1.4	0.829 0.5	0.765 0.6	0.760 1.6	0.663 1.2	0.609 1.4	0.605 2.6	0.551 1.6	0.481 2.9	0.461 3.4	0.468 2.9	0.543 -0.3
1.079 -0.3	1.113 1.0	1.066 1.0	0.000 0.0	0.959 0.6	0.967 1.8	0.832 0.7	0.769 0.7	0.763 1.4	0.668 0.8	0.614 1.7	0.614 3.0	0.520 2.3	0.000 0.0	0.447 4.2	0.448 4.1	0.533 0.0
1.037 -0.3	1.072 1.1	0.972 1.1	0.985 1.5	0.950 1.1	0.000 0.0	0.813 1.4	0.748 1.6	0.000 0.0	0.655 1.9	0.599 2.4	0.000 0.0	0.533 2.5	0.473 3.3	0.411 4.5	0.441 3.8	0.527 0.0
1.006 -0.1	1.082 0.6	0.973 1.2	0.929 1.5	0.908 0.8	0.913 1.1	0.812 0.9	0.757 1.1	0.749 1.5	0.659 1.5	0.607 1.7	0.594 2.2	0.514 2.7	0.455 3.6	0.422 4.1	0.462 3.4	0.535 0.6
1.012 0.3	0.906 0.4	0.852 0.1	0.816 0.5	0.785 0.3	0.751 0.7	0.708 0.2	0.665 0.3	0.624 1.0	0.579 0.9	0.536 1.0	0.495 2.1	0.450 2.2	0.409 2.7	0.385 3.0	0.411 2.7	0.579 -0.1

Table 8. Power distribution end errors %, for <4F> approximation SHM. UO2-2 Fuel assembly, 2D-problem

0.790 0.7	0.788 0.4	0.770 0.4	0.748 0.4	0.723 0.6	0.695 0.9	0.656 0.8	0.617 0.9	0.581 1.3	0.539 1.2	0.499 1.5	0.461 2.4	0.420 2.6	0.383 3.1	0.363 3.6	0.381 2.7	0.502 0.2
	0.830 -0.4	0.836 -0.5	0.829 -0.1	0.817 -0.6	0.809 -0.2	0.743 -0.4	0.698 -0.2	0.676 0.5	0.611 0.2	0.567 0.9	0.540 1.4	0.481 1.1	0.433 2.3	0.405 2.2	0.418 1.4	0.531 -0.2
		0.870 -0.5	0.902 -0.1	0.892 -0.5	0.000 0.0	0.800 0.0	0.751 0.3	0.000 0.0	0.659 0.8	0.610 1.2	0.000 0.0	0.530 1.4	0.479 2.4	0.432 1.9	0.433 1.1	0.537 -0.1
			0.000 0.0	0.897 -0.7	0.849 0.2	0.771 -0.4	0.724 -0.3	0.704 0.6	0.636 0.2	0.592 0.4	0.573 1.6	0.534 1.4	0.000 0.0	0.455 2.3	0.437 1.4	0.533 -0.2
				0.848 -1.0	0.823 0.1	0.751 -0.1	0.706 0.2	0.687 0.7	0.621 0.3	0.578 0.8	0.558 1.8	0.508 1.2	0.484 2.1	0.454 2.0	0.436 1.5	0.522 -0.5
					0.000 0.0	0.748 0.6	0.704 1.1	0.000 0.0	0.621 1.2	0.577 1.5	0.000 0.0	0.499 2.1	0.461 2.7	0.000 0.0	0.436 2.2	0.507 0.5
						0.688 0.2	0.650 0.4	0.634 1.4	0.574 0.9	0.533 0.9	0.512 2.3	0.457 1.9	0.422 2.1	0.413 3.0	0.405 1.7	0.484 0.3
0.615 0.8	Power for SHM calculation Error, % [Ref-SHM]*100./Ref						0.615 0.8	0.601 1.4	0.544 1.2	0.506 1.3	0.485 2.5	0.433 1.9	0.399 2.8	0.391 3.3	0.384 2.1	0.461 0.1
								0.000 0.0	0.532 2.1	0.494 2.4	0.000 0.0	0.423 3.0	0.390 3.4	0.000 0.0	0.375 2.6	0.437 0.5
									0.482 1.4	0.448 1.8	0.431 2.9	0.384 2.3	0.356 2.6	0.349 3.5	0.342 2.4	0.409 0.4
										0.417 2.1	0.400 3.2	0.359 2.8	0.332 3.1	0.325 3.4	0.319 2.6	0.381 0.7
											0.000 0.0	0.348 3.3	0.322 3.8	0.000 0.0	0.306 3.0	0.355 1.2
												0.317 2.6	0.302 3.3	0.285 3.3	0.273 3.1	0.325 1.1
													0.000 0.0	0.258 4.5	0.246 4.2	0.295 2.0
														0.231 4.9	0.227 4.4	0.272 1.9
															0.224 3.2	0.261 1.7
																0.284 0.9

Table 9. Power distribution end errors %, for <3F> approximation SHM. UO2-1 Fuel assembly, 3D-problem

2.216 -0.8	2.221 -0.9	2.231 -0.8	2.243 -0.8	2.250 -1.0	2.251 -0.9	2.205 -0.8	2.166 -1.0	2.139 -0.9	2.073 -0.8	2.016 -1.0	1.967 -1.1	1.874 -1.0	1.770 -0.9	1.649 -1.0	1.499 -1.0	1.292 -0.9
	2.233 -0.9	2.260 -1.0	2.299 -1.0	2.325 -1.0	2.395 -0.9	2.274 -0.8	2.233 -1.0	2.271 -1.0	2.138 -0.9	2.080 -1.0	2.093 -1.1	1.938 -1.0	1.816 -1.1	1.672 -1.1	1.508 -1.2	1.292 -0.9
		2.333 -0.7	2.470 -1.1	2.497 -1.0	0.000 0.0	2.410 -1.1	2.361 -1.0	0.000 0.0	2.261 -1.2	2.202 -1.3	0.000 0.0	2.083 -1.3	1.955 -1.2	1.730 -1.2	1.529 -1.2	1.297 -0.8
			0.000 0.0	2.524 -1.0	2.486 -1.2	2.317 -1.0	2.266 -0.8	2.315 -1.0	2.169 -1.0	2.118 -1.0	2.171 -1.3	2.102 -1.1	0.000 0.0	1.836 -1.2	1.559 -1.3	1.304 -0.9
				2.426 -0.9	2.457 -1.3	2.301 -1.0	2.252 -0.9	2.302 -1.0	2.156 -0.9	2.104 -1.1	2.146 -1.2	2.021 -1.2	1.996 -1.1	1.855 -1.2	1.579 -1.1	1.310 -1.0
					0.000 0.0	2.373 -1.1	2.326 -1.0	0.000 0.0	2.228 -1.3	2.170 -1.1	0.000 0.0	2.051 -1.3	1.965 -1.4	0.000 0.0	1.630 -1.1	1.314 -0.9
						2.238 -0.9	2.196 -0.9	2.248 -1.1	2.105 -0.9	2.049 -0.8	2.078 -1.1	1.923 -1.1	1.834 -1.1	1.792 -1.3	1.548 -1.1	1.288 -1.0
2.238 -0.9	Power for SHM calculation Error, % [Ref-SHM]*100./Ref						2.158 -0.8	2.208 -1.0	2.068 -0.9	2.013 -1.0	2.040 -1.1	1.885 -1.0	1.796 -0.9	1.758 -1.2	1.523 -1.1	1.269 -0.8
								0.000 0.0	2.119 -1.2	2.061 -1.1	0.000 0.0	1.931 -1.2	1.838 -1.2	0.000 0.0	1.555 -1.2	1.258 -0.9
									1.984 -1.0	1.932 -0.9	1.959 -1.0	1.811 -0.9	1.727 -0.8	1.693 -1.2	1.466 -0.9	1.222 -0.8
										1.883 -1.0	1.911 -1.0	1.770 -1.0	1.692 -1.1	1.654 -1.2	1.433 -1.0	1.195 -0.8
											0.000 0.0	1.811 -1.2	1.739 -1.3	0.000 0.0	1.453 -1.1	1.175 -0.9
												1.709 -1.0	1.692 -1.2	1.582 -1.1	1.352 -1.2	1.127 -0.9
													0.000 0.0	1.496 -1.2	1.277 -1.2	1.076 -1.0
														1.333 -0.9	1.188 -1.3	1.020 -1.0
															1.089 -0.8	0.959 -0.4
																0.879 -0.1

Table10. Power distribution end errors %, for <3F> approximation SHM. PU Fuel assembly, 3D-problem

1.323 -0.8	1.063 -0.1	0.940 -0.4	0.870 -0.6	0.819 -0.6	0.778 -1.1	0.717 -0.5	0.667 -0.1	0.626 -0.6	0.572 0.0	0.524 0.3	0.485 0.1	0.435 1.2	0.393 1.8	0.369 2.3	0.400 2.5	0.597 0.4
1.304 -0.7	1.337 0.4	1.166 0.3	1.095 -0.2	1.042 0.8	1.051 -0.4	0.912 0.8	0.844 0.6	0.843 -0.2	0.725 1.1	0.664 1.3	0.655 0.8	0.556 2.2	0.496 1.8	0.457 2.7	0.501 3.0	0.589 0.6
1.299 -0.7	1.313 0.4	1.169 0.8	1.184 -0.7	1.120 0.1	0.000 0.0	0.960 -1.1	0.877 -0.8	0.000 0.0	0.761 -0.5	0.691 -0.2	0.000 0.0	0.601 1.3	0.538 0.8	0.460 3.2	0.492 3.4	0.585 0.6
1.302 -0.7	1.328 0.3	1.269 -0.7	0.000 0.0	1.112 0.0	1.139 -1.0	0.944 1.5	0.867 1.5	0.879 -0.3	0.746 1.6	0.684 1.9	0.702 0.1	0.578 1.6	0.000 0.0	0.505 1.8	0.499 2.6	0.585 0.2
1.304 -0.7	1.345 0.9	1.287 0.1	1.178 0.3	1.125 1.8	1.089 -0.2	0.914 1.6	0.842 1.5	0.856 -0.4	0.724 1.9	0.663 2.0	0.675 0.1	0.592 3.4	0.527 2.0	0.505 2.6	0.505 3.4	0.585 0.2
1.308 -1.0	1.423 0.3	0.000 0.0	1.291 -0.7	1.148 -0.4	0.000 0.0	0.991 -0.5	0.906 -0.2	0.000 0.0	0.788 -0.5	0.713 0.4	0.000 0.0	0.616 0.7	0.571 1.1	0.000 0.0	0.537 2.3	0.586 0.2
1.281 -0.6	1.317 1.2	1.230 -0.4	1.129 1.7	1.021 1.5	1.043 -0.7	0.885 1.8	0.818 2.0	0.836 -0.3	0.705 2.0	0.645 2.2	0.651 0.2	0.544 3.1	0.504 3.1	0.479 1.6	0.496 3.6	0.578 0.8
1.263 -0.6	1.297 1.1	1.206 -0.5	1.105 1.7	1.002 1.8	1.023 -0.4	0.872 1.7	0.807 1.8	0.824 -0.7	0.696 2.2	0.637 2.3	0.641 0.4	0.536 3.2	0.495 3.4	0.471 2.2	0.490 3.4	0.572 0.5
1.253 -0.8	1.360 0.4	0.000 0.0	1.193 -0.1	1.078 -0.5	0.000 0.0	0.949 -0.5	0.871 -0.2	0.000 0.0	0.759 -0.2	0.687 0.4	0.000 0.0	0.585 0.7	0.532 1.7	0.000 0.0	0.518 2.5	0.571 0.3
1.218 -0.3	1.253 1.5	1.170 -0.7	1.071 1.6	0.972 1.7	0.996 -0.4	0.847 2.0	0.786 2.2	0.804 -0.1	0.679 2.3	0.621 2.5	0.627 0.5	0.524 3.4	0.484 3.5	0.461 1.9	0.479 4.0	0.560 0.8
1.192 -0.6	1.227 1.2	1.147 -0.4	1.056 1.6	0.958 1.9	0.979 -0.2	0.834 1.7	0.773 1.6	0.790 -0.1	0.669 2.1	0.613 2.4	0.618 0.8	0.518 3.3	0.480 3.6	0.456 2.3	0.474 4.4	0.553 0.6
1.173 -0.9	1.283 0.3	0.000 0.0	1.169 -0.6	1.044 0.0	0.000 0.0	0.909 -0.3	0.833 0.0	0.000 0.0	0.728 0.1	0.661 0.5	0.000 0.0	0.573 1.1	0.531 1.4	0.000 0.0	0.502 2.8	0.550 0.8
1.128 -0.5	1.172 1.2	1.131 0.4	1.034 0.7	0.993 2.0	0.970 -0.2	0.816 2.0	0.755 2.3	0.772 0.3	0.654 2.5	0.601 2.9	0.615 0.8	0.539 3.5	0.480 2.5	0.463 2.8	0.463 3.9	0.537 0.9
1.083 -0.4	1.118 0.7	1.081 0.0	0.000 0.0	0.956 0.6	0.986 -0.3	0.823 1.9	0.759 2.1	0.773 0.2	0.659 2.7	0.607 2.7	0.626 0.9	0.518 2.5	0.000 0.0	0.454 2.3	0.450 3.4	0.528 0.6
1.040 -0.5	1.073 0.9	0.973 1.3	0.998 -0.2	0.951 0.8	0.000 0.0	0.825 0.0	0.760 0.1	0.000 0.0	0.666 0.6	0.608 0.9	0.000 0.0	0.534 2.0	0.480 2.0	0.412 4.1	0.441 3.9	0.522 1.0
1.009 -0.2	1.082 0.9	0.975 1.0	0.934 0.7	0.901 1.7	0.918 0.5	0.804 1.8	0.751 1.9	0.754 0.8	0.654 2.5	0.603 2.7	0.596 1.9	0.509 3.2	0.456 3.1	0.422 4.0	0.461 3.5	0.529 1.4
1.018 -0.3	0.910 -0.1	0.855 -0.1	0.820 0.0	0.788 -0.1	0.757 -0.3	0.709 0.1	0.665 0.5	0.629 0.2	0.580 1.1	0.536 1.5	0.498 1.0	0.451 1.8	0.410 2.3	0.387 2.4	0.410 2.5	0.572 0.7

Table 11. Power distribution end errors %, for <3F> approximation SHM. UO2-2 Fuel assembly, 3D-problem

0.793 0.4	0.791 0.1	0.774 0.0	0.753 -0.3	0.728 -0.5	0.702 -0.3	0.659 0.2	0.620 0.5	0.586 0.4	0.541 0.7	0.501 1.0	0.465 1.1	0.423 1.7	0.387 2.1	0.365 2.5	0.381 2.4	0.494 2.0
	0.830 -0.3	0.835 -0.3	0.831 -0.6	0.814 -0.4	0.810 0.0	0.740 -0.1	0.696 0.1	0.678 0.3	0.609 0.7	0.564 0.7	0.541 1.4	0.479 1.6	0.434 2.0	0.405 2.2	0.416 2.2	0.521 1.5
		0.868 -0.5	0.904 -0.4	0.889 -0.4	0.000 0.0	0.802 -0.1	0.753 0.2	0.000 0.0	0.661 0.3	0.613 0.4	0.000 0.0	0.528 1.2	0.480 2.0	0.430 2.3	0.430 1.9	0.528 1.2
			0.000 0.0	0.893 -0.2	0.853 -0.1	0.767 0.2	0.720 0.2	0.705 0.6	0.632 0.9	0.587 1.3	0.574 1.1	0.531 1.8	0.000 0.0	0.455 1.9	0.435 2.0	0.523 1.5
				0.839 0.1	0.823 0.0	0.746 0.3	0.702 0.5	0.688 0.7	0.617 1.0	0.573 1.3	0.558 1.3	0.503 1.9	0.482 2.2	0.451 2.4	0.431 2.3	0.513 1.0
					0.000 0.0	0.748 0.5	0.705 0.8	0.000 0.0	0.622 1.1	0.577 1.4	0.000 0.0	0.499 2.1	0.463 2.3	0.000 0.0	0.433 2.6	0.498 1.5
						0.683 0.8	0.646 0.8	0.635 1.0	0.570 1.3	0.529 1.8	0.513 1.8	0.454 2.3	0.419 3.0	0.413 3.0	0.400 2.7	0.475 1.6
0.611 1.0	Power for SHM calculation Error, % [Ref-SHM]*100./Ref						0.611 1.0	0.601 1.3	0.540 1.6	0.502 1.6	0.485 1.7	0.430 2.3	0.397 3.1	0.391 2.9	0.380 2.9	0.452 1.9
								0.000 0.0	0.533 1.4	0.495 1.9	0.000 0.0	0.424 2.4	0.390 3.0	0.000 0.0	0.373 3.0	0.430 1.9
									0.479 1.8	0.445 2.5	0.431 2.5	0.382 2.7	0.353 3.3	0.348 3.6	0.338 3.4	0.401 2.3
										0.414 2.6	0.401 3.0	0.356 3.3	0.329 3.6	0.324 3.8	0.315 3.7	0.374 2.7
											0.000 0.0	0.348 3.2	0.322 3.9	0.000 0.0	0.303 3.9	0.348 3.1
												0.313 4.0	0.300 4.6	0.283 4.5	0.270 4.1	0.318 3.2
													0.000 0.0	0.258 4.6	0.245 4.0	0.289 3.8
														0.230 4.7	0.224 4.9	0.267 3.8
															0.221 4.4	0.256 4.1
																0.276 3.6

CONCLUSIONS

Resulting tables show:

1. The pin power results for the main part of the massif deviate from the reference values by not more than 2 percent. ste ping
2. The only exception is the region of the "pit" that is the region bounded some lines back one line from the reflector boundary. The error amounts to 5,3% for 3F approximation and 4,9% for 4F approximation used here.
3. Deviations from reference power distributions are nearly the same for both UO_2 Fuel-Clad and MOX Fuel-Clad including their boundaries.
4. Deviations in multiplication factors are 72 pcm for 3F- and 54 pcm for 4F-approximations.
5. Power distributions for 2D and 3D calculations have almost no deviation one from another.
6. It should be noted that error in multiplication factor estimate totals 1.2% and some tens percent of power distributions in MOX Fuel-Clad for homogenized cells and standard finite-difference equations.

We tried to analyze visible deflections of power distributions within the region of the “pit” and found three possible sources of those errors.

a) Our effective diffusion coefficient is slightly overestimated for the thermal groups due to G_2 – approximation used.
b) There is an error of the reference result due to the use of the M-C approach for the problem with $k_{eff}>>1$. Perhaps, difficulties arise from a selection problem of superfluous histories when there is the fast increase of a solution.

There is an error of the reference result that comes from small cell powers in the “pit”. Here cell powers are at least ten times smaller than powers of center cells.

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REFERENCES

1. .E.E.Lewis, M.A.Smith, N.Sulfanidis, G.Palmotti, T.A.Taiwo, R.N. Blomquist, “Benchmark specification for Deterministic 2-D/3-D MOX fuel assembly transport calculations without spatial homogenization (C5G7 MOX)”, **NEA/NSC/DOC(2001)4**
2. N.I.LALETIN,N.V.SULTANOV,V.F.BOYARINOV."Surface Harmonic and Surface Pseudo-Sources Methods."-Proceedings of International Conference on the Physics of Reactors,v.2,pp.XII-39-XII-48,April 23-27, Marsille, France, 1990.
3. N.I. Laletin, A.A. Kovalishin, "Even Approximations of Surface Pseudo-Sources Method for Reactor Lattice Effective Diffusion Coeffisient Computing", in Proc. 16th ICTT, Atlanta, USA, 1999.

4. N.I. Laletin, A.A. Kovalishin, ON APPLICABILITY OF THE <8F>-APPROXIMATION OF THE SURFACE HARMONICS METHOD FOR COMPUTING RBMK-TYPE REACTOR, in Proceedings of the PHYSOR 2000 ANS International Topical Meeting on Advances in Reactor Physics and Mathematics and Computation into the Next Millennium, May 7-12, 2000 Pittsburgh, Pennsylvania, USA